HOW MUCH OF A HAIRCUT?

OPTIONS-BASED STRUCTURAL MODELING OF DEFAULTED BOND RECOVERY RATES

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Originally Drafted: March 6, 2006
This Draft: August 17, 2006

Preliminary and Incomplete:
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Abstract

The present paper characterizes the problem of estimating recoveries on defaulted debt in a real options optimal stopping framework that takes account of put-call parity conditions embedded within corporate capital structures. The paper hypothesizes that an optimal stopping problem can theoretically explain a large amount of the variability in losses on defaulted corporate debt securities, and that augmenting this approach by estimating in a system of equations that accounts for put-call parity conditions adds considerable explanatory power. Empirical tests with a large number of corporate defaults confirm the usefulness of the approach. Moreover, the approach creates a powerful framework for analyzing investor behavior across the business cycle. Increased volatility, combined with time-varying net discount rates around business cycle turning points, can result in stakeholders waiting longer in search of additional returns before renegotiating the debt "haircut," that is, the reduction in face value of debt and/or increase in stated maturity necessary to resolve the default.

Defaults on corporate debt hurt firm creditors. Moreover, if defaulted debt imposes greater losses and longer emergence times during economic downturns, heightened risk aversion in credit markets may restrict new lending and create a drag on economic growth. In recent years, literature like Calomiris and Mason (1997, 2003) showed that investors often act rationally even during financial panics and literature such as Hart and Moore (1998) suggested that defaulted debt may be rationally priced. Hence, while academics, financiers, and regulators have long been interested in predicting the incidence of defaults, attention has more recently turned toward predicting losses incurred in default and the timing of ultimate emergence from default.[1]

Defaults and bankruptcies have long been of interest in finance, but the reasons for that interest have shifted over time. Corporate finance initially focused on bankruptcy costs as a means of justifying deviations from Modigliani and Miller's (1958) capital structure theory. Later work by, for instance, Hart and Moore (1998), suggested that defaulted debt may be part of optimal capital structures, meaning that defaulted debt could be priced at equilibrium conditions (as long as asymmetric information about the true condition of the firm can be overcome). Most recently, revisions to the Basel Capital Accord (Basel II) have built upon and institutionalized the equilibrium pricing approach, focusing attention on estimating loss-given-default (LGD) and using that as a means of establishing bank regulatory capital requirements for individual banks in developed countries.

Reduced form models estimated in the early empirical bankruptcy cost literature identified three primary determinants of bankruptcy costs: firm size, asset specificity, and industry performance (see, for instance, Alderson and Betker, 1995, 1996). As researchers have been driven, both by profit opportunities and regulatory concern, to improve models estimating default

[1] We refer throughout to emergence as the completion of the default, i.e., the date and price at which the firm begins again to repay its (potentially renegotiated or repriced) debt.

duration and the value of debt upon emergence (denoted here as "emergence value"), it has become increasingly important to move from reduced form models to structural models. An important step in that direction is the contribution of Shleifer and Vishny (1992), which suggested that there exists cyclicality in the returns on defaulted debt over the business cycle. Nonetheless, the three primary empirical determinants of bankruptcy costs, i.e., firm size, asset specificity, and industry performance, have yet to be fully incorporated in comprehensive structural models of recovery. Hence, there has recently been a great deal of work attempting to apply the empirically-demonstrated effects of firm size, asset specificity, and industry performance to structural models of default duration and emergence value.

Most of the existing structural work attempts to extend existing models of default (i.e., Merton, 1974) or debt value (i.e., Jarrow and Turnbull, 1995, Duffie and Singleton, 1999, and others). Much of that work, however, assumes a static external environment, so adjusting for industry performance and credit cycles has been difficult.

The present paper contributes to the default duration and emergence value literature by starting with a real options optimal stopping model of emerging from (or resolving) corporate financial distress.[2] The optimal stopping model is then estimated jointly for debt and equity in a system of equations to take advantage of information implicit in Black and Scholes (1973) and Merton (1974) put-call parity conditions embedded within firm capital structures (referred to hereafter as "BSM put-call parity"). In combining the optimal stopping and capital structure models, the present approach acknowledges that default events (i.e., nonpayment, formal declaration of default, and legal declaration of bankruptcy), and the resulting losses, are the

[2] See Dixit and Pindyck (1994) and Mason (2005) for discussion and specific applications of optimal stopping models to this problem within the real options framework. For a recent example of applying real options to other corporate finance problems see Carlson, Fisher, and Giammarino's (2006) model of long term underperformance of seasoned equity offerings.

results of joint decisions of owners and creditors (see, for instance, LoPucki and Whitford, 1990). In terms of options-based capital structure theory, low equity values (low values of the shareholders' long call position on the firm's assets) and low debt values (high absolute values of the short put on the firm's assets that is sold by creditors) cause a struggle for the control of firm assets among shareholders and creditors. Default events are one manifestation of that struggle. Necessary negotiations to adjust the terms of the options (particularly the exercise price and maturity on the put sold by creditors) serve to not only dictate the loss due to default based upon optimal stopping conditions for the struggle, but do so while maintaining put-call parity. The present paper argues, therefore, that the additional information included by modeling the conditions of optimal stopping while accounting for put-call parity can yield substantial insight into the ultimate emergence values of defaulted debt.

Testing the option-based structural model on Standard & Poor's data collected from the resolutions of over 1,000 defaulted corporate bonds from 1987-2003, the empirical models establish that the options parameters for an optimal stopping specification (namely, volatility and the discount rate – net of expected growth in the firm's assets during default) and necessary identification variables are statistically significant and obtain the theoretically correct signs. Furthermore, the most basic models, containing only the options parameters and the identification variables, explain up to 45% of the variation in emergence value of the defaulted debt. Parsimoniously adding several well-chosen control variables further increases the explanatory ability of the model to almost 60% of the variation in the emergence value of the debt. Additional models with ad hoc controls commonly used in the literature explain just under 80% of the emergence value of the debt.

While valuing defaulted debt using the real options optimal stopping model augmented with BSM put-call parity increases the power of LGD models, it also addresses two additional concerns of previous work in the field. First, prior work has questioned whether to estimate emergence value in terms of percent of par or dollar value.[3] In the present application, however, the optimal stopping theory itself dictates the form of the dependent variable (that is, the total return from the debt's market value at default to its value at emergence). Second, macroeconomic effects (in earlier parlance, "industry performance") are naturally incorporated into the optimal stopping approach as changes to the underlying volatility and net growth rate parameters that determine the values of the shareholder and creditor stopping options. Higher asset market volatility decreases creditors' willingness to wait during business cycle expansions and commensurately decreases both nominal and discounted recoveries (and therefore increases LGD). Hence, the present approach provides a parsimonious framework that can accommodate systematic variations in loss-given-default arising from fundamental business cycle conditions

The rest of the paper proceeds as follows: Section I provides the context of the analysis within the broader literature; Section II illustrates the hypothesized theoretical relationships that determine emergence value; Section III describes the data used in the empirical tests; Section IV presents empirical procedures and the results of those tests; and Section V summarizes and concludes.

[3] See, for instance, Guha and Sbuelz (2005).

I. Background

The literature on estimating Probability of Default (PD) and Loss-Given-Default (LGD) since the original Altman (1968) and related applications is too voluminous to cover completely.[4] The broad literature, however, has three fundamental foundations. The first is a PD foundation, which is the fundamental basis for bond ratings and commercially available applications like Z-score models and Moody's-KMV.[5] The second is a "bankruptcy cost" literature primarily stemming from analysis of violations of the Modigliani-Miller capital structure theorem in corporate finance. The third is an LGD literature originating primarily from rating agencies and banks to price financial instruments in primary and secondary markets.

The PD literature dates to the original Altman (1968) models of default. Those models typically regress key forward-looking financial variables on default events to estimate a set of regressors that can predict default out-of-sample.[6] As PD models sought to predict more effectively out-of-sample, they eventually evolved to a set of structural models based primarily upon variants of Merton (1974), which also forms the basis of the commercially available Moody's-KMV default model. Models in the PD literature, however, focus exclusively on measuring the probability of a discrete event.

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[4] Some of the more recent literature has built models in terms of Recovery Rate (RR) instead of LGD. In percent terms, researchers have defined the recovery rate on a defaulted security as: RR = 100 – LGD. It is important to further note that the timing of when the RR is measured is not always clear in the literature. Often, the literature refers to RR as the recovery rate at the time of default or some relatively short time period after default (e.g., the recovery rate based on the bond's market value 30 days after default). In the present analysis, the recovery rate is measured over the normally longer time interval that starts with either the default date or the last cash payment made by the debtor and ends when the debtor 'emerges' from default or bankruptcy. This broader view of the recovery rate period is more representative of the true optionality of the debt renegotiation process. Guo, Jarrow, and Zeng (2005) make a similar distinction and follow a similar approach.

[5] Note that we refer here only to PD for convenience. In practice, models are used to measure the determinants of a number of different discrete dependent variables, including, but not limited to, cash payment, default, and bankruptcy. A newer generation of models is used to predict more continuous phenomena like profitability, revenue, and servicing cost. Those models, however, are beyond the scope of the present work. See, for instance, Moody's-KMV and FairIsaac corporate literature for more information.

[6] See, for instance, Altman (1968), Ohlson (1980), Zmijeski (1984), Begley, Ming, and Watts (1996), Shumway (2001), Hillegeist, Keating, Cram, and Lundstedt (2002), Saretto (2004).

LGD and bankruptcy cost models, in contrast, are hybrid models of losses conditional on that discrete event. The bankruptcy cost literature began as a way to evaluate whether bankruptcy costs are of sufficient magnitude to justify observed deviations from Modigliani and Miller's (1958) capital structure theory.[7] Classic literature like Warner (1977) and Weiss (1990) was originally intended in that vein. Alderson and Betker (1995, 1996) summarize from that literature three major effects on bankruptcy costs: firm size, asset specificity, and industry performance. Variables that capture those effects provide the most powerful predictions of the time it takes to work out a bankruptcy, and are directly related to administrative costs, which are the primary measurable bankruptcy costs (in contrast to, say, agency costs) that might drive capital structure decisions.

As more data on events leading to firm distress became available, more academic interest arose in predicting losses on financial instruments accompanying those precursor events (events like withholding cash payment, formal default, and bankruptcy). As academics proceeded in that vein, the academic work began to approach practitioner research by major ratings agencies that estimated LGD in order to price financial instruments in both primary and secondary markets. Most recently, the practitioner models have achieved even greater importance as they have become the focus of the "internal models" approach for the Basel II regulatory capital framework for banks in developed countries worldwide. Literature like Covitz and Han (2004), Carey and Gordy (2004), and Hanson, Pesaran and Schuermann (2005) relates primarily to industry and regulatory concerns over more precisely measuring LGD.

The primary distinction between the bankruptcy cost and LGD literatures is the adaptation to systemic phenomena. That is, as the LGD models for ratings agencies and banks began to

[7] See Alderson and Betker (1995; 1996) for useful reviews of that literature.

search for macroeconomic environment effects (primarily to capture procyclicality with industry conditions and macroeconomic performance) in models of cash payments and defaults (instead of just bankruptcy), the resulting research became very similar to the literature estimating models of bankruptcy costs in corporate finance where industry performance interacts with firm size and asset specificity to determine bankruptcy costs.

Shleifer and Vishny (1992) was one of the first academic studies to link the bankruptcy cost models of corporate finance theory and the LGD models of practitioners by asserting the procyclicality with respect to industry conditions in both literatures. Following Shleifer and Vishny's (1992) revelation, academic researchers began to conduct comprehensive empirical investigations of aggregate recovery rates.[8] Nonetheless, unlike Merton (1974) in the PD literature, a more or less universally accepted structural model of LGD that can be used to guide empirical work has yet to be developed. That is not to say, however, that there have not been significant efforts to develop those structural models.

A good amount of work in developing structural models of LGD has attempted to branch directly off of known structural models of PD.[9] For instance, Frye (2000a, 2000b), builds upon Finger (1999) and Gordy (2000) in developing a structural model wherein defaults are driven by a single systematic factor: the state of the economy. The state of the economy is positively associated with Recovery Rate (RR) and negatively associated with PD, an observation consistent with the most recent U.S. bond market data. Frye's (2000a, 2000b) empirical analysis allows him to conclude that, in a severe economic downturn, Moody's bond database data on default prices suggest that bond RR's might decline 20–25 percentage points from their normal-

[8] See, for instance, Izvorski (1997), Hu and Perraudin (2002), Acharya, Bharath, and Srinivasan (2003), Altman, Brady, Resti, and Sironi (2006), Bris, Welch, and Zhu (2004).

[9] The reader is directed to Altman, Brady, Resti, and Sironi (2005) for a more comprehensive review of contemporary models.

year average. Loan recoveries, using Moody's Investors Service data for 98 senior secured bank loans, may decline by a similar amount. Note, however, that Frye uses only "default prices," which are the market prices of the debt instruments within 30 days of default. Since defaults, like bankruptcies, may take quite some time to resolve, Frye does not really estimate the *eventual* emergence value.

Of greater relevance to the present approach are the methodological contributions of Jarrow (2001) and Jokivuolle and Peura (2000). While Jokivuolle and Peura (2000) rely on an option pricing method to value defaulted debt, the firm's asset value does not determine the RR. Rather, Jokivuolle and Peura (2000) assume that equity value is unobservable in default. As a result of that assumption, Jokivuolle and Peura (2000) rely on a correlation parameter between the value of the firm and the value of the debt to infer the RR. Models like Bakshi, Madan, and Zhang (2001) also estimate those correlation parameters. Bakshi, Madan, and Zhang (2001), however, go one step further than Jokivuolle and Peura (2000) by allowing a flexible correlation between the risk-free rate, the default probability, and the recovery rate. Forcing recovery rates to be negatively associated with default probability, their empirical results show that, on average, a 4% worsening in the (risk-neutral) hazard rate is associated with a 1% decline in (risk-neutral) recovery rates.

Jarrow's (2001) suggested structural estimation method, on the other hand, assumes that equity and debt together form the basis of firm value, obviating the need for the correlation parameter. Hence, the model builds upon Merton (1974) by allowing the PD component of conditional LGD to be based upon fundamental equity valuation parameters, while debt value is either priced at par (if not in default) or at the remainder of firm value (if the firm is in default). Jarrow (2001) does not value the equity, per se, however, but merely includes equity value in the specification so that debt value becomes a residual of firm assets minus equity. Guo, Jarrow, and Zeng (2005) is even closer in spirit to the present approach, but uses a more computationally complex stopping model and contains no empirical tests.

The present approach therefore builds upon Jarrow (2001) and Guo, Jarrow, and Zeng (2005) by treating the RR as a function of the capital structure of the firm, but, like Jokivuolle and Peura (2000), uses option valuation methods to estimate the equilibrium capital structure using the BSM characterization.[10]

It is shown below that the present method has three chief advantages over previous work. First, the approach is a structural method similar in principle to that of Jarrow (2001) and Guo, Jarrow, and Zeng (2005), so that it uses information in both equity and debt prices to estimate RR. Second, the approach accommodates the macroeconomic environment through option valuation parameters: changes to volatility and discount rates influence the value of the options and put-call parity, and hence the value of equity and debt. Third, while literature like Guha and Sbuelz (2005) suggests that different model results obtain for dollar value of recovery and recovery measured as a percentage of par, and therefore question the appropriate recovery measure, the present model answers that debate. The decision variable in the present specification, the growth in debt value during default, comes directly from the continuous-time real options optimal stopping specification used to characterize the equity and debt options.

In summary, this paper argues that the theoretical approach offered in Section II is more intuitive, less complex, and more powerful than existing reduced form or structural recovery models. Furthermore, the empirical results in Section IV demonstrate that not only does the model make intuitive sense, but even the simplest model estimated, including option valuation

[10] At its heart, the entire line of inquiry cited above is firmly rooted in Geske (1979), which viewed risky debt as a compound option and that the shareholders' choice to default is a linear sequence of finite options.

parameters for volatility and the net discount rate (as well as the relevant identification variables), explains up to 45% of the variation in emergence value of the defaulted debt. Parsimoniously adding several well-chosen control variables further increases the explanatory ability of the model to around 60% of the variation in the emergence value of the debt. Models with ad hoc controls frequently included in the literature can explain upwards of 80% of the emergence value of the debt.

II. Theoretical Approach

The present theoretical approach brings together two key elements of previous work. First, the present framework estimates returns during default as an optimal stopping problem (see, for instance, Dixit and Pindyck 1994 for the theory and Mason 2005 for relevant applications). Second, the approach augments information about optimal debt values with the relative value of other financial instruments in the capital structure, particularly common equity. Within the BSM capital structure framework, equity and debt values are related to one another via the put-call parity relation. Hence, modeling equity values simultaneously with debt values can improve the explanatory ability of the model of defaulted debt. The present section steps through the links in the underlying models that provide testable implications for the empirical work that follows.

A. Real Options Specification of Optimal Stopping Time

Since the option to emerge is open-ended and irreversible, the present application is one of an optimal shutdown option, which itself is a form of a perpetual timing option.[11] Shareholders

[11] The optimal stopping model used in the present approach is similar in nature to the model of Guo, Jarrow, and Zeng (2005), which, itself, is based upon work by He, Wang, and Yan (1992). Those models, however, specify solutions that decompose accessible and inaccessible stopping times. The real options approach used here transforms that decomposition into higher or lower target values for the firm's debt, V^*, desired for stopping and, in our

and creditors emerge from default when asset values look promising, i.e., there is no more growth to be economically gained by creditors before irreversible emergence from default but shareholders can still compel a "haircut" (a reduction in the face value of debt and/or an increased in the debt's stated maturity) to creditors.

Mason (2005) shows the theoretical solution for such an optimal timing option and empirically estimates the effects of the options valuation parameters for bankrupt firm liquidations (the timing of the final put by the creditors). The theoretical approach from Mason (2005) is described in detail in Appendix A. Two things are important about the solutions to the class of real options presented in Appendix A: 1) the comparative statics are generally the same as those of Black-Scholes for a European option on a stock index with a continuous dividend yield, and 2) the dependent variable defining the option value in that class of problems is specified as the growth in value during the timing period, relative to the initial investment price.

The primary difference between a limited-term option and an open-ended perpetual option is the characterization of the option's value. In the real options variant, value maximization is applied to some characterization of $V_t – I$, where V_t is the value of the debt at time t after default and I is the value of debt at default. With stochastic growth in the underlying V_t, an optimal maturity date, $T*$, is not relevant. Rather, exercise is contingent on V_t exceeding some critical value, V^*, such that stopping is optimal once $V_t \geq V^*$. or, expressed in terms of yields, $\Delta V^* \geq (V^* - I) / I$.

ΔV^*, in turn, is a function of the expected net discount rate and volatility. Appendix A demonstrates formally that ΔV^* for a long option rises in response to greater volatility and declines in response to higher net discount rates (i.e., net of the expected price growth in the

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opinion, creates more straightforward testable implications for the empirical tests that are the main point of the present manuscript. All the results presented here generalize to using time as a dependent variable.

underlying asset's value, similar to how a European option on a stock index with a continuous dividend yield uses the discount rate minus the continuously compounded dividend payment rate). That means that if opportunity costs are high, target ΔV^* will be low, and if expected volatility is high, target ΔV^* will be high.[12] If ΔV^* is an unacceptably low value, i.e., one that values the firms in the range of economic insolvency, creditors will shut down the firm immediately and liquidate the remaining assets following the process described in Mason (2005).[13]

B. Put-Call Parity View of Capital Structure

The present exercise applies the real options model described above to a defaulted debt valuation process. One of the main problems of defaulted debt valuation is that since the principal value of the bonds in question is subject to renegotiation, it is not clear what face value is due in the future. Furthermore, since the relevant maturity and coupon of the bonds in question is subject to renegotiation (or, in the worst case, maturity and coupon are irrelevant), it is not clear what kind of interest rate risk is embedded in bond prices during default. Last, the valuation of defaulted debt often needs to extend beyond mere bond prices, valuing instead the packages of bonds, cash, and equity that are often extended to creditors in a distressed workout.

It is important to realize, however, that the debt valuation process that is being settled in order to resolve default is not settled in isolation. Rather, the debt value is part of an overarching capital structure, the entirety of which must be revalued in order to support the firm's emergence

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[12] It is easy to see that if growth is deterministic, V^* can be mapped into an optimal T^*. The Appendix formally presents that deterministic solution, as well as the more meaningful stochastic variation.

[13] As in Mason (2005), all the results presented here generalize to a feasible real options specification using T^* as the dependent variable instead of V^*. Mason (2005) shows, however, that the appropriate characterization of the critical value V^* substantially improves the explanatory ability of the model.

from default. Since the emergence process itself is being viewed as an options-based problem, it is useful to look at the influence of capital structure through the options perspective as well. The rest of this section shows that viewing the value of defaulted debt in the context of the entire capital structure can theoretically contribute substantial additional explanatory power to the real options model of debt valuation described above.

BSM developed the common way to view capital structures as a portfolio of options. The BSM capital structure approach is most succinctly presented in terms of simple European options. [14] Following Hull (2006, p. 214), consider a company with no asymmetric information and no agency costs that has assets that are financed with zero-coupon bonds and equity. Suppose that the bonds mature in T=5 years at which time a principal payment of K is required. The company pays no dividends. If the assets are worth more than K in 5 years, the equity holders choose to repay the bondholders. If the assets are worth less than K, the equity holders choose to declare bankruptcy and the bondholders end up owning the company.

The minimum value of the equity in 5 years is characterized as $max(A_T\text{-}K, 0)$, where A_T is the value of the company's assets at that time. Hence, the equity holders have a 5-year European call option on the assets of the company with a strike price of K. The bondholders get $min(A_T, K)$ in 5 years, which is the same as $K\text{-}max(K\text{-} A_T, 0)$. The bondholders have therefore given the equity holders the right to sell the company's assets to them for K in 5 years. The bonds are thus worth the present value of K minus the value of a 5-year European put option on the assets with a strike price of K.

To summarize, if c and p are the value of the call and put options, respectively, then:

[14] While the present approach uses the BSM model of European put-call parity in the capital structure to reinforce the optimal stopping framework, the problem is really more aptly characterized as put-call parity with American options. Interestingly, Merton (1973) discussed the issue of an option's value as the option's maturity approaches an infinite length but did not formulate a specific closed-end solution for pricing a perpetual put option. The present results hold without loss of generality to a specification of put-call parity for American options.

$$\textit{Value of equity} = V_e = c \tag{1}$$

$$\textit{Value of debt} = V_d = PV(K)\text{-}p \tag{2}$$

which conform to put-call parity. Denoting the value of the assets of the company today by A_0, firm value must equal the sum of the value of the equity and the value of the debt, so that

$$A_0 = c + [PV(K) – p] \tag{3}$$

Rearranging this equation, results in:

$$PV(K) - p = A_0 - c \tag{4}$$

which is the put-call parity result for call and put options on the assets of the firm with a strike price, *K*, equal to the face value of the firm's debt and common maturity, *T*.

In the context of the BSM capital structure, the problem of default is characterized by a wide difference between the values of *c* and *p*. With a value near zero, the shareholders' call option, $c = V_e$, is deep out of the money. If *c* has little value, then $PV(K)\text{-}p = V_d$ has a value near A_0. Hence, if *PV(K)* is above A_0 (the firm is economically insolvent) then *-p*, at a very large negative value, is deep in the money, which means that shareholders have a strong incentive to put A_0 to the creditors.

The decreasing value of *c* relative to the increasing (absolute) value of *-p* creates the fundamental struggle for control over A_0: creditors can seize A_0 by shutting down the firm, while shareholders can seize A_0 (at least in the short term) by filing for bankruptcy. The classic solution to that struggle is to negotiate a reduction in the value of *PV(K)*, i.e., take a "haircut." There are two ways to reduce the value of *PV(K)*: reduce the value of *K* (the par value of the debt) and increase the value of *T* (the maturity of the debt). No matter which method is chosen, reducing *PV(K)*, by definition, decreases the moneyness of *-p* and increases the moneyness of *c*, thereby reducing the incentives for the shareholders to exercise the put against the creditors through bankruptcy.[15]

C. Applying the Real Options Model to Valuing Defaulted Debt

Now place the real options estimation problem in the context of the BSM notation. Let $PV(K)\text{-}p=V_d$ and center time on the default, letting t_0 be the moment of default and t_E be the moment of emergence. Then V_{d0} is the dollar value of debt at default and V_{dE} is the dollar value of debt at emergence from default. Using that notation, the single-equation real options models in the empirical section estimate the effects of underlying net discount rates and volatility on the dependent variable denoted as the "debt yield during default," where the yield is defined as $\Delta V_d^* = (V_{dE} - V_{d0}) / V_{d0}$. If the real options optimal stopping time model is valid in the context of defaulted debt valuation, then the single-equation model should explain a substantial amount of variation in ΔV_d^*.

Because $\Delta V_d^*=\Delta A_0\text{-}\Delta V_e^*$, the real options model of optimal stopping time applies to V_e as well. Intuitively, creditors and shareholders have incentives that create unique solutions for ΔV_d^* (which is equal to $\Delta A_0\text{-}\Delta V_e^*$). Assuming, again, no asymmetric information or agency costs, creditors will not accept a haircut that is based on less than ΔV_d^* or they will lose substantial upside potential to V_d that is expected to accrue soon after the default is resolved. Conversely, shareholders will not wait until after ΔV_d^* occurs to emerge from default because that means giving up some longer-term growth to creditors unnecessarily. Hence, both creditors and shareholders have an incentive to agree and act immediately upon a unique ΔV_d^*.

[15] Nonetheless, *PV(K)* can never be reduced by an amount greater than the transactions costs of liquidating the firm's assets or the creditors will go ahead and do just that.

If put-call parity constrains the capital structure and if equity values (being more uniform and more actively quoted) convey information that debt values do not, it is useful to specify an equity valuation model to be estimated simultaneously with the debt model. A real options equity valuation model can be specified by relating the effects of underlying net discount rates and volatility on the dependent variable "equity yield during default," defined as $\Delta V_e^* = (V_{eE} - V_{e0}) / V_{e0}$. If augmenting the real options model of debt valuation with the real options model of *equity* valuation is a superior approach to analyzing debt values in isolation, then the multiple equation specifications should explain substantially more variation in ΔV_d^* than their single-equation counterparts.

D. Summary

In summary, the optimal stopping model characterizes the problem of estimating recovery rates as an equilibrium between two sets of stakeholders (shareholders and creditors) with competing interests. Each can be thought of as determining their debt renegotiation strategies based upon their expectations for firm performance. Based on that view, the firm emerges from default (or bankruptcy) when asset values look promising to both parties, that is, creditors see no more growth to profitably gain before irreversible emergence, but shareholders can still compel a haircut to creditors in financial reorganization.

Also embedded in that relationship, stakeholders hold two sides of an option: shareholders have a call on the assets of the firm; creditors have sold a put on the assets of the firm. Hence, a further empirical implication for put-call parity arises because information from one side of the option relates to the other. Information on equity values can therefore be useful for estimating defaulted debt values at emergence in an optimal stopping time model of default.

The next section, Section III, first describes the data and empirical approach. Section IV demonstrates that the resulting theoretical framework, firmly grounded in accepted theory of the value of the firm, explains a high percentage of variation in debt yields during default. The next two sections, therefore, demonstrate that the approach just described bridges not only a theoretical gap in the literature, but also an empirical gap between models of firm value and bankruptcy costs.

III. Empirical Methods and Data

The empirical work below develops two stylized equations – one representing the debt yields during default, ΔV_d*, and another representing equity yields during default, ΔV_e*, that estimate emergence values of corporate debt both individually and jointly The debt and equity yields during default are determined from the discount rate (δ) and volatility (σ) variables. The equations are used to test three empirical conjectures. First, if the real options optimal stopping time model is valid, those two option-related variables should explain a large proportion of the variation in the data sample's debt yields during default.[16] Second, if the BSM put-call parity approach adds value to the specification, jointly estimating debt and equity yields during default should explain a significantly higher proportion of variation in debt yield during default. Third, if the approach is superior to other approaches in the literature the signs and statistical significance of the discount rate (δ) and volatility (σ) variables should be robust to including a panoply of

[16] It should be noted that we do not expect our theoretical model to explain 100% of the variation in defaulted bonds' recovery rates. Our main focus is to demonstrate that option-related variables such as δ and σ are significant determinants of recovery rates. Nevertheless, as is presented below, we find that not only are δ and σ both highly significant but also that the overall explanatory power of the model, as measured by adjusted R^2, is quite high (i.e., surpassing 80% in the most detailed specification).

control variables used in previous empirical studies. The empirical results that follow support all three of those conjectures.

Because real world capital structures can be quite complex, the present application does not impose strict endogeneity between the creditor and shareholder return specifications. Econometrically, therefore, there is no problem with simultaneity that would necessitate two- or three-stage least squares estimation.[17] Nonetheless, any structural system requires exogenous variables in each equation in order for the system to be identified. Hence, identifying variables that determine equity but not debt returns (or vice versa) are included in all joint specifications. The resulting system of equations is solved by seemingly unrelated regression.

The models are specified as:

$$\Delta V^*_e = \text{Equity Return during Default} = f(\delta, \sigma, \sigma{*}RECESSION, \delta{*}RECESSION, PERATIO) \quad (5)$$

and

$$\Delta V^*_d = \text{Debt Return during Default} = f(\delta, \sigma, \sigma{*}RECESSION, \delta{*}RECESSION, LEVERAGE, ORIGINALMATURITY, SECURED, Controls) \quad (6)$$

where, *Controls* refers to 13 additional *ad hoc* control variables taken from prior literature.

Like Mason (2005), the present models utilize equity returns to generate proxies for the underlying net discount rate and asset volatility that, in turn, translate into option value. In all models, the net discount rate, δ, is the CRSP value-weighted stock index annual stock return minus the firm's compounded return for the annual period prior to the event. Volatility, σ, is the

[17] Models estimated allowing target equity, ΔV_e*, and debt, ΔV_d*, to enter each other's models endogenously in a proper simultaneous equations framework obtain identical signs and statistical significance to the results that follow. To conserve space, we focus on the simpler model that does not explicitly account for simultaneity.

standard deviation of monthly firm returns for the annual period prior to the event.[18] All models are restricted to observations where the firm's equity is listed for at least half the period that the firm is in default.

Mason (2005) hypothesizes that it is possible that the creditors' willingness to accept a lower *K* and longer *T* will vary with the business cycle. During cyclical expansion, creditors will want a higher *K* and shorter *T* and, conversely, during cyclical contraction, creditors will be more likely to accept a lower *K* and a longer *T*. As in Mason (2005), therefore, interactions of δ and σ with associated business cycle peaks and troughs allow the signs on the options valuation parameters to switch depending on the participants' views of the business cycle.

The variables used to test the three conjectures outlined at the beginning of this section are described in detail below. The basic models explain debt yields during default with only the options-theoretic parameters, δ and σ, and relevant identification variables. Multiple-equation specifications used to account for BSM put-call parity in the capital structure estimate the effects of those options-theoretic parameters, δ and σ, and relevant identification variables in a joint specification explaining both debt and equity yields during default. Models with hoc control variables include variables commonly thought to affect recovery rates (variously defined) some of which could be expected to have high correlations with the options-theoretic parameters, δ and σ. No matter what specification is used, the options-theoretic parameters, δ and σ, obtain the correct signs and explain a large amount of the variation in debt yields during default.

[18] Results below are robust to a wide variety of different specifications on these variables. See Section IV, Empirical Tests, for more detail.

A. Dependent Variables

The present analysis relies primarily upon the S&P LossStats database, the same database used in Carey and Gordy (2004) and Acharya, Bharath, and Srinivasan (2003).[19] Those data cover defaults on corporate bonds and subsequent recoveries from 1987 to 2003. The primary data set is supplemented with data from S&P Compustat, the Center for Research in Security Prices (CRSP), and the National Bureau of Economic Research (NBER). Variable names, summary statistics, and variable definitions and correlations are presented in Table I, Panel A.

The time path of defaults and recoveries is illustrated in Figure 1. Defaults peak in business cycle recessions and subside in other periods. Of key concern to the present analysis are different concepts of emergence value that are used as dependent variables in the specifications below. For the debt models that follow, the specifications measure effects of four dependent variables derived from two different definitions of debt recoveries.[20] The specification is also robust to defining the dependent variables solely as time in terms of event days (as demonstrated in Appendix B).

Properly testing emergence value in the options-theoretic framework described earlier requires defining some notion of ΔV_d* as a dependent variable. The S&P LossStats database provides three possible ΔV_d* variables to choose from, each presented in both nominal and discounted terms: Trading Price at Emergence, Settlement Price, and Liquidity Event Price. As explained in the LossStats user guide:

> *The recovery value of an instrument can be determined by using the trading price or market value of the pre-petition debt instruments upon a bankruptcy emergence. This method is similar to the commonly used "30 days*

[19] Neither of those papers, however, uses testable implications from structural models for formally guide the empirical approaches.

[20] Other definitions of debt recoveries were estimated, but not included in the present manuscript. The manuscript presents only those specifications with the most observations. The results presented are robust to the various definitions. Results of alternative specifications are available upon request.

after default" method, except that the trading price is measured at emergence instead of 30-days after default.[21] *Of the three methodologies, this one is the most readily available since most debt instruments continue to trade during bankruptcy proceedings.*

Settlement Pricing includes the prices of instruments exchanged for the debt in emergence.

The settlement pricing includes the earliest public market values of the new instruments a debt holder receives in exchange for the pre-petition instruments. It is similar to the trading price method, except that it is applied to the new (settlement) instrument(s) instead of the old (pre-petition) instrument. The settlement pricing may comprise more than one instrument, whereas each instrument would be valued and summed together to arrive at the recovery value. Part of the settlement may involve cash, that can be valued immediately, but when part of the settlement involves common stock or debt instruments, the trading prices may not be immediately available.

Liquidity Event Pricing uses not the earliest available prices on the exchanged instruments, as does Settlement Pricing, but the prices of those instruments at the emergence date: "The liquidity event price is the final cash value of the new instruments which were acquired in exchange for the pre-petition instrument," at the date the restructuring is confirmed (*LossStats User Guide)*.

LossStats also includes a fourth category of recovery, the "S&P Preferred Method," that selects one type of recovery (i.e., Trading Price at Emergence, Settlement Price, or Liquidity Event Price) that is deemed to be the most representative in terms of the individual emergence event.

Each of the three (four, including the S&P Preferred method) are presented in both nominal and discounted terms. For discounting, LossStats discounts each ultimate recovery back to the last date that cash was paid on the bond. The discounting takes place using the standard present value formula using the pre-petition interest rate – which is the coupon rate for bonds or the effective interest rate for non-fixed instruments – as the discount rate.[22]

[21] An internal S&P study finds that trading prices may not accurately reflect the financial strength of the debtor upon emergence.

[22] See the *LossStats User Guide* for additional details.

The analysis below highlights results using the S&P Preferred Method and the Settlement Price methods in both nominal and discounted terms. In general, the Trading Price and Liquidity Event Price methods yield too few observations to provide meaningful empirical estimates.

S&P Preferred and the Settlement Price recoveries still need to be converted to a return concept that more closely mimics ΔV_d* in the real options theory. In the present analysis, that is accomplished by assuming that the debt is trading at par on the day of default. The difference between par and recovery is converted to an annualized return basis for use as a dependent variable.[23] The resulting variables are RECOVERYNSP (Nominal S&P Preferred Method), RECOVERYDSP (Discounted S&P Preferred Method), RECOVERYNST (Nominal Settlement Price Method), and RECOVERYDST (Discounted Settlement Price Method). Summary statistics for each are presented in Table I, Panel A.

Figure 2 presents histograms of the distribution of the different recovery methods. Of course, since each is measured relative to par, recoveries yield primarily negative rates of return. Those histograms illustrate that some twenty percent of events yield a nominal gain over par at recovery, although in discounted terms less than one percent of events yield a gain from par at recovery. That result is not surprising given the amount of time it takes to resolve defaults, illustrated in Figure B1 in the Appendix. Roughly fifteen to eighteen percent of events yield a 100% loss from par, whether measured on a nominal or discount basis. Roughly ten to twenty percent of event outcomes illustrate zero gain or loss from par on a nominal or discount basis.

[23] Specifications were also estimated with a more properly specified variable that used t+30 price instead of par to better measure actual appreciation in default, that is, the benefits of waiting to emerge. Unfortunately, t+30 data observations are limited, so the sample dropped from 663 to 132 in the S&P Preferred model specifications, and 447 to 99 in the Settlement Price specifications. Furthermore, S&P warns that the t+30 data (the same data used by Frye) are extremely noisy and not very useful. Nonetheless, the individual specifications with the t+30 data maintain adjusted R^2 statistics of 22% to 26% and the joint specifications for the Nominal and Discounted S&P Preferred Method and Nominal Settlement Price specifications maintain adjusted R^2 statistics of 20% to 30%. The joint specification for Discounted Settlement Price, with only 54 observations, returns a negative adjusted R^2. The three meaningful specifications maintain the signs and statistical significance of all the options-theoretic parameters save the business cycle expansion discount rate.

Hence, about fifty to sixty percent of events (depending on the recovery variable used) have intermediate outcomes involving some sort of loss measured on either a nominal or discount basis.

The equity return variables are constructed to measure the stock return during the default event window using the period from the last cash payment to emergence and the period from default to emergence (too few observations remained to implement the bankruptcy-to-emergence window). FIRMEQUITYRETURNC2R measures equity returns from last cash payment to emergence, FIRMEQUITYRETURND2R measures equity returns from default to emergence. All data used to construct the equity returns are from the CRSP daily stock returns database. Figure 3 illustrates that while some equity investors face negative returns in the event period, others sometimes experience dramatic gains.

Figure 4 illustrates the tendency for firms to be delisted during default. Eleven percent of the firms in our sample are listed less than 50% of the days they spend in default. In order to produce meaningful estimates of equity returns in the models that follow, therefore, those eleven percent of firms are omitted from the specifications that follow.

B. Options Volatility and Net Discount Rate Independent Variables

The net discount rate (δ) is specified according to the real options theory as the discount rate minus the expected growth rate of the asset.[24] As in Mason (2005), the δ used in the models below is computed as the CRSP Value-weighted index return minus the firm's own return over the year prior to the event. Volatility (σ) is specified as the traditional volatility of the returns,

[24] This is similar to an American option on an index that pays dividends, where the net discount rate is the discount rate minus the continuous dividend payment rate.

using the standard deviation of the firm's (monthly) returns for the year prior to the event.[25] As can be seen by the average levels of δ and σ in Table I, Panel A, the net discount rates and volatilities associated with the firms in our sample are quite high (with average annualized values of 0.869 and 0.296, respectively). It should be noted that the high average δ is the result of subtracting a

-0.8119 average firm stock return from a +0.0575 average market return, leading to a +0.8694 average value for the net discount rate.

Again, following Mason (2005), net discount rate (δ) and volatility (σ) are interacted with a dummy variable characterizing whether or not the default occurs during business cycle recessions. Recession periods are take from the NBER business cycle database, and in the present application include July 1, 1981 to November 30, 1982; July 1, 1990 to March 1991; and March 1, 2001 to November 30, 2001. The variables δ*RECESSION and σ*RECESSION indicate growth and volatility for defaults occurring during NBER recessions.

C. Other Explanatory Variables

The models below include a number of other independent control variables. Those are broadly characterized into two classes, identifying variables and other ad hoc control variables that are commonly used in the related literature.

Identifying variables are used in the jointly estimated specifications to separate econometrically the equity and debt models. Identifying variables, like instrumental variables, are exogenous variables that explain either debt or equity value, but not both. While the present specification is not a simultaneous equations application, the identifying restrictions help make

[25] Specifications using five-year Treasury note risk-free rates and dividend growth and company betas as alternative discount rate and volatility proxies also work, but at the cost of a smaller sample size.

sure the specifications avoid problems of observational equivalence on the margin by nonetheless satisfying standard rank and order conditions.

The debt models are always presented with three identifying variables. Those variables are LEVERAGE (proportion of debt principal senior to the defaulted debt), ORIGINALMATURITY (original maturity of the defaulted debt), and SECURED (dummy variable equal to one if the defaulted debt is collateralized). On average, about 22% of the debt of companies defaulting is senior to the instruments in default. The proportion of debt principal senior to defaulted debt is referred to as LEVERAGE because that is the effective leverage relating to the particular debt issue in default – junior debt is irrelevant to that issue and can be treated the same as equity for purposes of valuation. LEVERAGE is therefore expected to be negatively related to recovery. ORIGINALMATURITY of the defaulted instruments is, on average, about nine years. The expected sign on ORIGINALMATURITY is hard to determine *a priori*. The effect could be positive, reflecting an increased propensity to wait for value to accumulate within the original maturity of the instrument. Alternatively, the sign on ORIGINALMATURITY may be negative, reflecting higher interest rate risk on longer maturity obligations. About 39% of the defaulted instruments are SECURED. Collateral security is expected to generally increase the recovery.

The equity models contain a single identifying variable, PERATIO (company price-earnings ratio at the time of the event). PERATIO averages -0.19 times actual earnings for the sample of defaulting firms. Nonetheless, the range on PERATIO runs from a minimum of -18.2 to 25.2, reflecting the diversity in firm earnings potentials at default.

The identifying variables were chosen based on the combination of their robust statistical significance in the debt and equity specifications and their primary relevance to debt and equity returns. As for statistical significance, the debt instruments are not typically statistically significant in the equity specifications and vice versa. As for primary relevance, PERATIO is commonly viewed as a proxy for the market's growth expectations specifically associated with the firm. Since that path of future earnings is a primary component of stock prices, PERATIO is therefore expected to be primarily related to equity returns rather than debt returns.

While it is harder to justify the relevance of LEVERAGE, ORIGINALMATURITY, and SECURED to debt values strictly on the grounds of financial theory, these variables are contract-specific characteristics that are based upon the conditions of the firm at the date the debt was issued. Hence, they necessarily relate primarily to the individual debt issue's seniority, and hence value, relative to the rest of the capital structure in a backward-looking fashion. (See Acharya, Bharath, and Srinivasan 2003 for more on the economic and statistical significance of these contract-specific variables.)

The identifying variables are meant to be exogenous to either the creditor and shareholder option values, that is, the returns on creditor and shareholder investments over the period of emergence. The ad hoc controls, however, are specified with little in the way of exogeneity assumptions. They are simply meant to test the robustness of the model to a variety of sources of additional explanatory power common in related literature as well as various sources of possible collinearity. It is important to point out that the objective of adding these sets of variables is not to obtain a parsimonious or econometrically valid or efficient predictive model, but to stress the robustness of the options valuations parameter signs and significance. Correlations among the entire set of dependent and independent variables are presented in Table I, Panel B.

To examine the above issue, one can generate some *a priori* expectations of the signs on each of the variables. CRSPVWC2R (CRSP value-weighted index return from last cash payment to emergence) and CRSPVWD2R (CRSP value-weighted index return from default to emergence) are both expected to relate positively to recovery because they are representative of overall financial market conditions during the emergence period. Nonetheless, the two are probably highly correlated, resulting in some multicollinearity in the specification.

BONDRATING (the company's numerical bond rating at the time of the event) rises as bond ratings deteriorate, so the expected sign is negative. DIVADJRFR (the current dividend-adjusted risk-free rate using U.S. 5-yr Treasury Note yield to maturity), DIVYLD (the company dividend yield at the time of the event), and INDPERATIO (the industry P/E ratio at the time of the event), represent determinants of firm profitability, and are therefore expected to be positive. 5YRTREASYTM (the U.S. 5-yr Treasury Note yield to maturity) is a risk-free discount rate that represents an alternative measure of the creditors' opportunity cost of capital, so the sign (like that of δ) is expected to be negative. Nonetheless, 5YRTREASYTM will most likely be correlated with δ, introducing collinearity in to the specification.

INDBETA (the industry average beta at the time of the event) and FIRMBETA (the company beta at the time of the event), as indicators of the underlying sensitivity of returns to systematic risk are expected to be negatively associated with recoveries, *ceteris paribus*. DEBTPRIORITY (the priority ranking of debt prior to default based on legal contract, using an inverse scale with 1 being highest priority), is expected to be negatively associated with recovery. $PRICE_{t+30}$ (the price of the debt 30 days after default as percent of par), is expected to be positively associated with recovery. MATDEF (the remaining maturity of debt at the time of the event) is expected to be positively related to recovery, *ceteris paribus*. Although MATDEF is expected to be somewhat correlated with ORIGINALMATURITY, the correlation will be reduced somewhat due to differences in the amount of time between issue and default. That same difference also accounts for the different expectations of the signs on MATDEF and ORIGINALMATURITY. PERCENTJUNIOR (the percent of debt principal that is junior to defaulted debt), being an indicator of seniority, should be positively related to recovery. PERCENTJUNIOR will also be correlated with LEVERAGE, introducing yet another source of collinearity to the specification. That correlation biases against the model stability we find below, where including PERCENTJUNIOR indeed sometimes affects the statistical significance of LEVERAGE, but it never affects the signs and significance of the option variables.

The variables LEVERAGE, ORIGINALMATURITY, SECURED, BONDRATING, DEBTPRIORITY, $PRICE_{t+30}$, MATDEF, and PERCENTJUNIOR are from S&P's LossStats database. The variables CRSPVWC2R, CRSPVWD2R, and 5YRTREASYTM are from CRSP. PERATIO, DIVADJRFR, DIVYLD, INDBETA, FIRMBETA, and INDPERATIO are from Compustat. Since the union of the Compustat accounting variables and the recovery rates based on S&P debt prices around the time of default have the most limited data availability, the specifications in Tables 6 and 7 that rely on those variables have the smallest sample sizes.[26]

IV. Empirical Tests

The empirical tests below measure not only the explanatory ability of the options-theoretic debt valuation model parameters, but also the additional explanatory ability contributed by jointly estimating the debt and equity models together to account for put-call parity conditions. Section IV.A below presents estimated coefficients from models of debt emergence value alone

[26] Our sample sizes are similar to those reported in Acharya et al. (2003), who use the same data set and also match against Compustat. However, Acharya et al. (2003) focuses on debt prices either around the default date (i.e., 30 days after default) or at emergence. For the S&P LossStats database, the choice of whether to use the t + 30 debt price or the debt price at emergence matters because there are significantly fewer observations available for the t + 30 debt price. For example, in Table 10 of Acharya et al. (2003), when the debt prices based on t + 30 days after default are used, the sample size is between 165 and 212 observations whereas the sample size increases to a range of 395-609 when debt prices at the time of emergence are used. Our results exhibit similar changes in sample size depending on whether or not our calculations employ the t + 30 debt prices.

to illustrate the signs and significance of the estimated coefficients and to provide a benchmark for comparison to the full BSM put-call parity system. Section IV.B shows the estimated coefficients from the BSM put-call parity system of equations.

In each section, estimates are presented for the several definitions of $\Delta V_d{*}$ offered above: RECOVERYNSP, RECOVERYDSP, RECOVERYNST, and RECOVERYDST. Equity models in the put-call parity system of equations are estimated using FIRMEQUITYRETURNC2R.[27] All the models obtain similar results, suggesting the approach is robust to a variety of specifications.

Most importantly, the models demonstrate that the BSM capital structure approach to defaulted debt valuation substantially increases the power of LGD models. By themselves, the option parameters for volatility, σ, and net discount rate, δ, and the three necessary identifying variables explain up to 45% of the variation in emergence value of the defaulted debt. Parsimoniously adding several well-chosen control variables further increases the explanatory ability of the model to just under 60% of the variation in the emergence value of the debt. Models with further ad hoc controls can explain around 80% of the emergence value of the debt.[28]

[27] Models were also estimated using FIRMEQUITYRETURND2R. The results that follow are robust to the choice of the equity returns timing.
[28] Five important notes should be made about robustness: First, monthly CRSP data produces the same results as the daily data. Second, additional proxies were tested throughout the empirical work, including Compustat data on firm and industry returns. The CRSP-based measures included here produce the same results with more observations. Third, the models also tested robustness to one-, three-, and five-year windows for the discount rate and volatility measures. The results below are robust to the time period of these discount rate and volatility measures. Fourth, specifications using five-year Treasury note risk-free rates minus dividend growth and company betas as alternative discount rate and volatility proxies also work, but at the cost of a smaller sample size (because they rely on Compustat rather than CRSP). Fifth, and as shown below, all results are robust to the dependent variable used to measure recovery, that is, the annualized growth in various S&P recovery measures or event days.

A. Individual Debt Models

Table II presents results from estimating the debt recovery model individually using the dependent variables designed to approximate the theoretically-derived ΔV_d*: RECOVERYNSP, RECOVERYDSP, RECOVERYNST, and RECOVERYDST. The S&P Preferred models use 663 observations, while the Settlement Price models use 447.

The options-theoretic parameters in all four specifications in Table II obtain the appropriate signs for a short put. That is, increased volatility, σ, is associated with lower target debt yields during default (by making the short put position more negative and thus reducing target ΔV_d*) and higher discount rates, δ, are associated with higher target debt yields during default (by making the short put position more positive and thus increasing target ΔV_d*) during business cycle expansions. The opposite effects obtain in business cycle contractions. All the options-theoretic parameters are statistically significant at conventional thresholds.

Among the identifying variables, LEVERAGE is negative and statistically significant at the 1% level, while the ORIGINALMATURITY is usually negative and statistically significant at the 1% level, and the dummy variable for SECURED debt is positive and statistically significant at the 1% level.

The explanatory power of the individually estimated debt recovery models is in the 29% to 35% range. Those values are more than three times that of the best performing event days dependent variable models demonstrated in Appendix B. This improvement comes about because the event days dependent variables are only loose proxies for more properly specified recovery-related dependent variables, ΔV_d*. As Mason (2005) points out, however, those event days dependent variables can be useful in analyzing what are commonly referred to as “feasible real options” specifications where proper recovery data does not yet exist.

In summary, the individually estimated debt models yield appropriate signs and statistical significance for the options-theoretic parameters, as well as solid explanatory ability for the models. But the capital structure theory expounded earlier suggests that allowing the value of equity to further influence the value of debt within the put-call parity framework can enhance the explanatory ability of the model. Hence, the next section estimates equity and debt values jointly and shows the added power of the approach.

B. Jointly Estimated Debt and Equity Models

The models in this section test whether coefficients from jointly estimating equity and debt yields are consistent with the put-call parity view of defaulted debt valuation advanced in earlier sections. As specified in Equations (5) and (6) of the previous section, the dependent variables in the equity and debt models are the yields on equity and debt realized during the period of default: RECOVERYNSP; RECOVERYDSP; RECOVERYNST; and RECOVERYDST for debt (ΔV_d*) and FIRMEQUITYRETURNC2R for equity (ΔV_e*) (the results are robust to the choice of the combination of debt and equity return measures).

Table III presents results from estimating the debt recovery model jointly with equity returns. Because some defaulted firms are delisted for substantial periods of time and others are not tracked by CRSP, the sample sizes decrease somewhat compared to the individual debt models presented earlier. Here, the S&P Preferred models use 438 observations, while the Settlement Price models use 288. Adjusted R^2 statistics on the equity models run from 3.4% to 4.7%, while those on the debt models (the ones of primary concern) increase from the 29%-35% range to the 34%-44% range.

The options-theoretic parameters in all four debt specifications in Table III again obtain the appropriate signs for a short put. That is, greater volatility is associated with lower debt yields during default and higher discount rates with higher debt yields during default in business cycle expansions. Again, the opposite effects obtain during business cycle contractions. All the options-theoretic parameters are statistically significant at conventional thresholds, and the LEVERAGE and SECURED identifying variables are again negative and positive, respectively, as well as statistically significant.

The options-theoretic parameters in all four equity specifications in Table III obtain the appropriate signs for a long call. Increased volatility, σ, is associated with higher equity returns during default and higher discount rates, δ, with lower equity returns during default in business cycle expansions. As before, the opposite effects obtain in business cycle contractions. All the options-theoretic parameters are statistically significant at conventional thresholds. The PERATIO identifying variable is positive and statistically significant at conventional levels in all but Columns C and D, where it remains positive nonetheless.

In summary, the fundamentally-derived BSM put-call parity method is able to explain up to 44% of the variation in debt yieldd during default without any additional controls. Furthermore, and most importantly, the BSM put-call parity framework is a structural model derived directly from both optimal stopping conditions and fundamental theories of firm value, yet adjusted R^2 statistics of the put-call parity models compare favorably with *un*adjusted R^2 statistics reported in ad hoc models like those of Acharya, Bharath, and Srinivasan (2003), Carey and Gordy (2004), and others. Hence, the next section explores whether adding similar ad hoc control variables can further increase the explanatory ability of the present structural model or whether those variables are merely correlates with the structural parameters tested here.

C. Increasing the Explanatory Ability of the Models and Testing Robustness

As benchmark references, other papers that measure determinants of recovery rates achieve explanatory abilities of up to almost 50% for Carey and Gordy (2004) (using recoveries discounted by the risk-free rate), up to 68% (although most are in the 50% range) for some of the models in Acharya, Bharath, and Srinivasan (2003) (using defaulted debt prices discounted by the high-yield bond rate), and up to 40% for Covitz and Han (2004) (using recovery rates at default).[29]

Tables 4 and 5 add the control variables described in section III.C to the specification in order to test robustness and maximize explanatory ability. Each table adds progressively more variables to the debt model to examine how far additional ad hoc control variables can push the optimal stopping put-call parity approach without upsetting the signs and statistical significance of the previously estimated options-theoretic parameter coefficients. It is important to point out that the objective in the specifications in Tables 4 and 5 is not to obtain a parsimonious or econometrically valid or efficient predictive model, but to stress the robustness of the options valuations parameter signs and significance.

Table IV shows the results of adding nine additional variables to the debt model: CRSPVWC2R, CRSPVWD2R, BONDRATING, DIVADJRFR, DIVYLD, INDPERATIO, DEBTPRIORITY, MATDEF, PERCENTJUNIOR. As noted earlier, sample size drops when the LossStats database is matched with CRSP and/or Compustat. Hence, the S&P Preferred models use 254 observations, while the Settlement Price models use 167.

[29] Note that although most models use a large number of regressors, the literature typically reports only raw, not adjusted, R^2 statistics.

Most, but not all, of the ad hoc variables are statistically significant. Signs on the variables, although not necessarily statistical significance, are consistent across specifications. CRSPVWC2R, DIVADJRFR, DIVYLD, INDPERATIO, and PERCENTJUNIOR all have a positive effect on return on debt yields during default, while CRSPVWD2R, BONDRATING, DEBTPRIORITY, and MATDEF all have a negative effect on return on debt yields during default.

The adjusted R^2 statistics of both the debt and equity models again increase significantly in Table IV with the additional ad hoc control variables. Adjusted R^2 statistics on the equity models run from about 6% to 8%, while those on the debt models (the ones of primary concern) run from 50% to 55%.

The options-theoretic parameters in all four debt specifications in Table IV maintain the appropriate signs for a short put even in the presence of the ad hoc controls. That is, increased volatility, σ, is associated with lower debt yields during default and higher discount rates, δ, with higher debt yields during default in business cycle expansions (the opposite effects obtain in business cycle contractions). The options-theoretic parameters in the equity model obtain the appropriate signs for a long call position. All the options-theoretic parameters are statistically significant at conventional thresholds.

Table V shows the results of adding a total of thirteen additional variables (the variables from Table IV as well as 5YRTREASYTM, INDBETA, FIRMBETA, and $PRICE_{t+30}$) to the original debt model. As noted earlier, sample size drops when the LossStats database is matched with CRSP and/or Compustat. Hence, the S&P Preferred models use 113 observations, while the Settlement Price models use 74.

Here, far fewer of the ad hoc variables are statistically significant, most likely due to high collinearity between combinations of these variables. Recall, however, that the objective in the specifications in Table V is not to obtain a parsimonious or econometrically valid or efficient predictive model, but to stress the robustness of the options valuations parameter signs and significance. Nevertheless, signs and statistical significance on the variables are consistent across specifications in Table V. DIVADJRFR, INDPERATIO, and $PRICE_{t+30}$ all have a positive statistically significant effect on debt yields during default, while SECURED, BONDRATING, and DEBTPRIORITY all have a negative statistically significant effect on debt yields during default.

The adjusted R^2 statistics of both the debt and equity models again increase significantly in Table V with the additional ad hoc control variables. Adjusted R^2 statistics on the equity models run from about 20% to 24%, while those on the debt models (the ones of primary concern) run from 75% to 80%.

The options-theoretic parameters in all four debt specifications in Table V obtain the appropriate signs for a short put even in the presence of the ad hoc controls. As seen previously, increased volatility, σ, is associated with lower debt yields during default and higher discount rates, δ, with higher debt yields during default in business cycle expansions (the opposite effects obtain in business cycle contractions). Once again, the options-theoretic parameters in the equity model obtain the appropriate signs for a long call. All the options-theoretic parameters in the debt models are statistically significant at conventional thresholds.

In summary, the options-theoretic parameters explain a significant amount of the variation in debt returns during default, and their effects remain significant despite the inclusion of numerous potentially confounding ad hoc control variables. Furthermore, the addition of even highly correlated ad hoc controls can increase explanatory power of the models to levels above those found in the previous literature while retaining the rigor and importance of a fundamentally derived set of structural parameters.

V. Summary and Conclusions

In summary, the present paper characterizes the problem of estimating recoveries on defaulted debt in a real options optimal stopping specification where information from equity prices enters through the BSM put-call parity relation within the firm's capital structure. According to the optimal stopping specification, defaulted debt is resolved when asset values look promising (so the put held by shareholders will not be reset at an inappropriately low strike price), but shareholders can still compel a "haircut" to creditors in the financial reorganization. Conceiving of debt and equity as two sides of the same put-call parity relationship suggests that equity prices can provide information on less liquid bond prices during periods of default.

The main contribution of the paper is that the options-theoretic approach to analyzing the determinants of emergence values for corporate debt successfully bridges theoretical and empirical gaps between models of firm value and default costs. Empirical tests with a large number of corporate bond defaults confirm the usefulness of the options-theoretic approach, whether in the form of the creditor option estimated in isolation or the creditor and shareholder options estimated jointly. The empirical models explain a high percentage of variation in debt emergence values, and do so in a structural model that is grounded in an accepted theory of the value of the firm.

Moreover, the options-theoretic view creates a powerful framework in which to analyze investor behavior across the business cycle as suggested by Shleifer and Vishny (1992), providing justification and testable implications for rational delay. Increased volatility, combined with varying discount rates (net of expected growth) around business cycle turning points, can result in stakeholders waiting to seek additional returns before settling on the renegotiated options parameters K and T (the face value of the debt and its maturity) necessary to resolve the default. As in Mason (2005), therefore, the dynamics of the model provide a clear mechanism that can promote prolonged illiquidity and business cycle persistence, and may contain seeds of financial market contagion in some asset sectors. For the moment, however, those macroeconomic extensions are left for future research.

Appendix A: Dynamics of Stochastic Real Put Option[30]

The problem at hand is what haircut can the shareholders impose upon the creditors in default. The solution is most easily demonstrated from the creditor perspective. As in Mason (2005), in the event that the shareholders exercise their put the creditors will then obtain a perpetual put on the assets of the firm. If creditors can be convinced that the emergence value is better than the present value of that perpetual put they will obtain in bankruptcy, they can be persuaded to restructure the debt and resolve the default. As in the Mason (2005) liquidation problem, therefore, creditors can be persuaded to emerge when asset growth is stable and further waiting is disadvantageous.

Assume the creditors do not face any incentive or agency problems and that there is one uniform portfolio to be put. Then following Dixit and Pindyck (1994), let V equal the current market value of assets to be put. Assume V follows a geometric Brownian motion process such that:

$$\delta V = \alpha V \delta t + \sigma V \delta z, \tag{A1}$$

where α is a drift parameter, σ is the variance, and δz is the increment of a Wiener process. Equation (A1) implies that the current value of the assets is known, but future values are lognormally distributed with a variance that grows linearly with the time horizon.

The creditors' emergence opportunity is equivalent to a perpetual put option. Therefore the decision to emerge is equivalent to deciding when to exercise that option.[31] Denote the value of the option to emerge as *F(V)*. The creditors choose the optimal time to exercise such that *F(V)* is maximized. Let I denote the amount of creditor claims, i.e., the value of the debt at default. Then

[30] This section follows closely from Mason (2005).
[31] An American option can be thought of as a variant of the perpetual option that is forced to exercise at a limit date. The perpetual option, however, has no such limit date so the exercise needs to be derived from a fundamental limit on the option value. It may be useful to bear in mind the results for a Black-Scholes model of the value of an option on an equity index that pays dividends through the following discussion.

the payoff from resolving at any time t is $V_t - I$, and at any time t the creditors' problem is one of maximizing the expected present value:

$$F(V) = \max \mathrm{E}\left[(V_t - I)e^{-\rho T} \right], \tag{A2}$$

where E is the expectation operator, T is the (unknown) future exercise date, ρ is the discount rate, and the maximization is subject to (A1) for V. It is important to assume that the drift parameter α in (A1) remains less than the discount rate ρ. Otherwise waiting longer would always be the dominant strategy and no optimum exercise time would exist. Hence, if α is allowed to vary across the business cycle creditors would be expected to emerge faster during a cyclical contraction and slower during an expansion.

The following two sections present two different solutions to the creditors' problem. A deterministic solution demonstrates that, even in the absence of uncertainty, there may exist value to the creditors from delaying liquidation. Then, a stochastic case is used to illustrate important comparative statics implications that are tested in the paper.

A. Deterministic Solution

Suppose σ in equation (A1) is zero. Then $V(t)=V_0e^{\alpha t}$ so that, given some current V the value of the emergence opportunity, assuming the creditors emerge at some arbitrary future time T, is:

$$F(V) = (Ve^{\alpha T} - I)e^{-\rho T}. \tag{A3}$$

Suppose $\alpha \leq 0$. Then $V(t)$ will remain constant or decline over time, implying that it is clearly optimal to emerge immediately.

A more interesting result arises when $0 < \alpha < \rho$. Then $F(V) > 0$ even if $V < I$ in the present period because V will *eventually* exceed I. This eventuality arises because although the future value of the initial investment held until T decays at $e^{-\rho T}$, the value of assets to be liquidated decays at the slower rate of $e^{-(\rho-\alpha)T}$.

How long will the creditors wait? Maximizing (A3) with respect to T yields the first order condition:

$$T^* = \max\left\{\frac{1}{\alpha}\log\left[\frac{\rho I}{(\rho-\alpha)V}\right], 0\right\}, \tag{A4}$$

so that if $V < \dfrac{\rho}{\rho-\alpha} I$, $T^* > 0$. Growth in V creates value to waiting and increases the value of the creditors' irreversible emergence opportunity.

B. Stochastic Solution

Now suppose $\sigma > 0$. Again, the creditors face an optimal stopping problem in continuous time. However, since V now evolves stochastically the creditors can no longer derive an optimal emergence time T^*. Instead, the emergence rule will comprise a critical value V^* such that emergence is optimal once $V \geq V^*$. Comparative statics demonstrate that both growth ($\alpha > 0$) and uncertainty ($\sigma > 0$) can create value to waiting and thereby prolong emergence.

The stochastic problem may be solved by dynamic programming. Without loss of generality, assume that the assets under liquidation yield no cash flows up to time T. Then in the continuation region $V < V^*$, the Bellman equation is:

$$\rho F \delta t = \mathrm{E}(\delta F) \,. \tag{A5}$$

Expand δF using Ito's Lemma to obtain:

$$\delta F = F'(V)\delta V + \frac{1}{2}F''(V)\delta V^2 \,. \tag{A6}$$

Substituting (A1) for δV in expression (A6) (noting that $\mathrm{E}\delta z = 0$) yields:

$$\mathrm{E}[\delta F] = \alpha V F'(V)\delta t + \frac{1}{2}\sigma^2 V^2 F''(V)\delta t\,, \tag{A7}$$

which can be substituted into (A6) to obtain the revised Bellman equation:

$$\frac{1}{2}\sigma^2 V^2 F''(V) + \alpha V F'(V) - \rho F = 0\,. \tag{A8}$$

Optimal V* is determined by solving (A8) subject to three boundary conditions. First,

$$F(0) = 0\,, \tag{A9}$$

restricts the payoff such that if *V* goes to zero, the option to invest is worthless. Next,

$$F(V^*) = V^* + I\,, \tag{A10}$$

restricts $F(V^*)$ to equal the investment *I* plus the value of the option V^*. Last,

$$F'(V^*) = 1\,, \tag{A11}$$

restricts *F(V)* to be a smooth continuous function in the region surrounding the emergence value V^*.

The optimal emergence value V^* is obtained by solving (A8) subject to the boundary conditions (A9), (A10), and (A11). Equation (A9) suggests the solution must take the form:

$$F(V) = AV^{\beta_1}\,, \tag{A12}$$

where *A* is a constant to be determined and $\beta_1 > 1$ is a known constant whose value depends on the parameters σ, ρ, and α in the differential equation (A8).

The remaining boundary conditions may be used to solve for the two remaining unknowns, the constant *A* and the critical emergence threshold V^*. Substitute (A12) into (A10) and (A11) to obtain:

$$V^* = \frac{\beta_1}{\beta_1 - 1} I\,, \tag{A13}$$

and

$$A = \frac{V^* - I}{(V^*)^{\beta_1}} = \frac{(\beta_1 - 1)^{\beta_1 - 1}}{\beta_1^{\beta_1} I^{\beta_1 - 1}}. \tag{A14}$$

The function AV^{β_1} solves equation (A8) provided β_1 is a root of the quadratic:

$$\frac{1}{2}\sigma^2 \beta(\beta - 1) + \alpha\beta - \rho = 0. \tag{A15}$$

The two roots of this quadratic are:

$$\beta_1 = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2\rho}{\sigma^2}} > 1,$$

and

$$\beta_2 = \frac{1}{2} - \frac{\alpha}{\sigma^2} - \sqrt{\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2\rho}{\sigma^2}} < 1.$$

The two roots suggest that general solution may be written as $F(V) = A_1 V^{\beta_1} + A_2 V^{\beta_2}$, but boundary condition (A9) restricts $A_2 = 0$, leaving the solution as that suggested in (A12).

The quadratic expression in general and β_1 in particular are functions of σ, ρ, and α, the parameters for which comparative statics are desired. To illustrate how the root β_1 responds to a change in σ, differentiate the quadratic expression totally. Let Q represent the expression in (A15) and β represent β_1 so that the differential with respect to σ can be written:

$$\frac{\partial Q}{\partial \beta}\frac{\partial \beta}{\partial \sigma} + \frac{\partial Q}{\partial \sigma} = 0.$$

Since α and β are both greater than zero:

$$\frac{\partial Q}{\partial \beta} > 0.$$

Then:

$$\frac{\partial Q}{\partial \sigma} = \sigma\beta(\beta - 1) > 0,$$

so it must be that:

$$\frac{\partial\beta}{\partial\sigma} < 0.$$

Thus as σ increases, β decreases and V^* increases so that the greater the uncertainty over future values of *V*, the larger the return the creditors will seek before irreversibly resolving the default.

Because V^* depends not only on the asset price growth α and the discount rate ρ, but on the *difference* between the two, their effects are examined with a slight modification. Let $\delta = \rho - \alpha$ and assume $\delta > 0$. Then the quadratic (A15) becomes:

$$\frac{1}{2}\sigma^2\beta(\beta-1) + (\rho-\delta)\beta - \rho = 0\,, \qquad \text{(A16)}$$

with root β_1:

$$\beta_1 = \frac{1}{2} - \frac{(\rho-\delta)}{\sigma^2} + \sqrt{\left(\frac{(\rho-\delta)}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2\rho}{\sigma^2}} > 1\,.$$

For δ, the differential is then:

$$\underset{(+)}{\frac{\partial Q}{\partial\beta}}\,\underset{(?)}{\frac{\partial\beta}{\partial\delta}} + \underset{(-)}{\frac{\partial Q}{\partial\delta}} = 0\,,$$

so it must be that

$$\frac{\partial\beta}{\partial\delta} > 0.$$

As δ increases, β increases and V^* decreases so that the greater the discount rate-price growth spread (the higher the opportunity cost to creditors relative to asset price growth), the smaller the return the creditors will seek before irreversibly resolving the default.

In summary, the stochastic model optimal emergence value *V** rises in response to greater volatility and declines in response to higher discount rate-price growth spreads.

Appendix B: Robustness of the Specification to Time-based Dependent Variables

Mason (2005) shows that the event days dependent variables are useful to testing the general effectiveness of the options-based specifications because those dependent variables and the theoretically proper ΔV^* dependent variables are positively correlated. Similar results are demonstrated here. The event days dependent variables, however, do not help predict any notion of loss given default.

Event days are days that the company spends in some characterization of default. The three different characterizations measured are: DAYSC2R (days from last cash payment to emergence); DAYSD2R (days from default to emergence); and DAYSB2R (days from bankruptcy to emergence).

DAYSC2R begins on the date when the last cash interest payment was made. According to S&P, for bonds that date is usually around six months before the instrument default date. The date when the last cash payment was made is important because the company may have information at that date that the market does not have.

DAYSD2R begins on the first date the company missed a scheduled interest payment. Each instrument issued by a company could have a different default date. That date may or may not be associated with a ratings action by a ratings agency.

DAYSB2R begins on the date the company files for bankruptcy, either voluntarily or non-voluntarily. (In the case of distressed exchanges, the bankruptcy date and the emergence date are both the date on which the distressed exchange was completed.)

Emergence date is the date at which the bankruptcy judge releases the company from bankruptcy protection or, in the absence of bankruptcy, the firm resumes debt payments at the negotiated settlement value. In the case of distressed exchanges, the bankruptcy date and the emergence date are both the date on which the distressed exchange was completed. The emergence date is also the date when pre-petition holders receive their settlement securities.

On average, it takes firms 572 days to move from last cash payment to emergence, 474 days to move from default to emergence, and 304 days to move from bankruptcy to emergence. Figure B1 presents histograms that illustrate the distribution of movement through those states. It is clear in Figure B1 that time spent in bankruptcy is small relative to the total amount of time spent in distress, that is, time from last cash payment to emergence.

Estimates of time in default, DAYSC2R, DAYSD2R, and DAYSB2R obtain similar results, suggesting the approach is robust to a variety of specifications. Furthermore, the models again show that the BSM capital structure approach to defaulted debt valuation substantially increases the power of LGD models.

Table BI presents results from estimating the debt recovery model individually using the timing of different default event concepts as dependent variables. The dependent variables presented in Table BI are DAYSC2R, DAYSD2R, and DAYSB2R. Greater volatility is associated with faster emergence while higher net discount rates are associated with slower emergences during business cycle expansions in all three of the specifications in Table BI. The opposite effects obtain in business cycle contractions. Since faster emergence is correlated with less return on investment, the signs are appropriate for the value of the short put held by the creditor. The effects of the options-theoretic variables are statistically significant in the DAYSC2R and the DAYSD2R specifications, but not for the DAYSB2R dependent variable (although almost two-thirds of the observations are lost implementing the DAYSB2R measure because relatively few default events ultimately result in formal bankruptcy proceedings).

Among the identification variables, LEVERAGE is positive and statistically significant at the 5% level, ORIGINALMATURITY is positive and statistically significant at the 1% level, and SECURED is negative and statistically insignificant. Adjusted R^2 statistics for the DAYSC2R and DAYSD2R dependent variables are 8% and 7% respectively.

Table BII presents results analogous to Table BI in that the debt-related return dependent variables are DAYSC2R and DAYSD2R. Column A includes DAYSC2R and FIRMEQUITYRETURNC2R; Column B includes DAYSD2R and FIRMEQUITYRETURNC2R; Column C includes DAYSD2R and FIRMEQUITYRETURND2R. Note that all models lose about 200 observations due to the availability of data on PERATIO in the equity model. The adjusted R^2 statistics are about the same magnitude as corresponding models in Table BI.

Like Table BI, the options-theoretic parameters of the debt models reported in Table BII obtain the correct sign and are statistically significant. The identifying variable for ORIGINALMATURITY in the debt model is always positive and statistically significant.

The options parameters in the equity specification yield insight into the validity of the put-call parity approach. The equity models in Columns A and B obtain signs on the options parameters during a business cycle expansion that are appropriate for the long call position associated with the payoff profile for the shareholders. The identifying variable, PERATIO, is statistically significant in both specifications and positive, suggesting that PERATIO may proxy for expected growth.

Although the equity model in Column C performs less well using the period from default to emergence for both the debt and equity models, it still obtains appropriate signs and significance on the options parameters during recession and the identification variable.

Again, the main point of the present exercise is to show that the event days dependent variables are useful to testing the general effectiveness of the options-based specifications because those dependent variables and the theoretically proper ΔV^* dependent variables are positively correlated. The event days dependent variables, however, do not help predict any notion of loss given default.

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Figure 1. Number of Default Events per Year

Number of default events is from Standard & Poor's LossStats™ Database, containing data on 1,808 corporate bond defaults between 1987 and 2003.

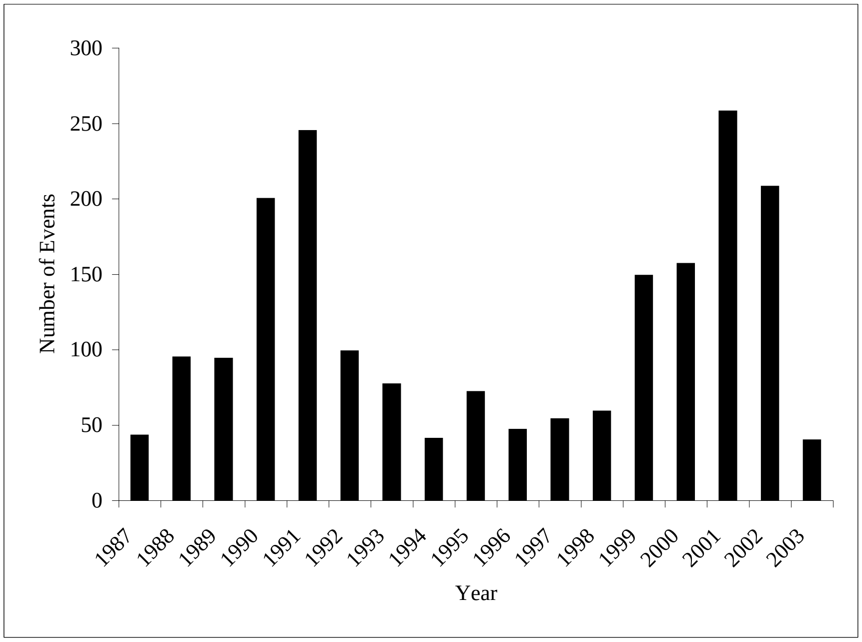

Figure 2. Distribution of Debt Yields during Default

Debt recoveries are from Standard & Poor's LossStats™ Database. Yields are annualized for comparison across different default durations. RECOVERYNSP is the S&P Preferred recovery estimate (amount the Trading Price, Settlement Price, and Liquidity Event Price estimation methods) expressed in nominal terms. RECOVERYDSP is the S&P Preferred recovery estimate expressed in present-value discounted terms. RECOVERYNST is the S&P Settlement Price recovery estimate expressed in nominal terms. RECOVERYDST is the S&P Settlement Price recovery estimate expressed in present-value discounted terms. All present value discounts are computed using the original face coupon rate.

Figure 2.A. Recovery at Resolution, RECOVERYNSP

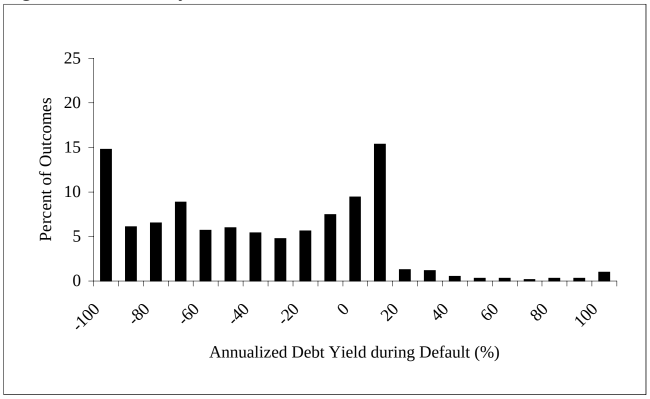

Figure 2.C. Recovery at Resolution, RECOVERYNST

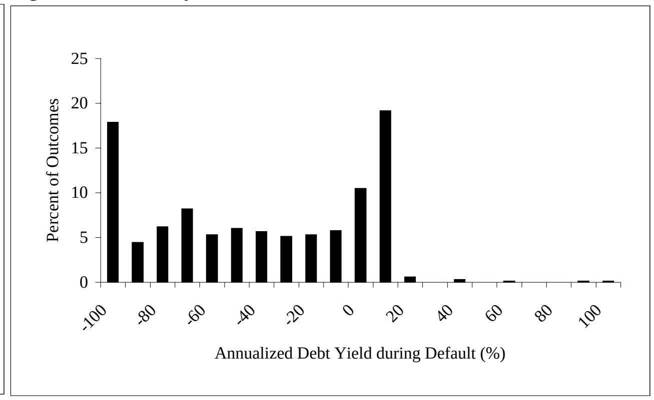

Figure 2.B. Recovery at Resolution, RECOVERYDSP

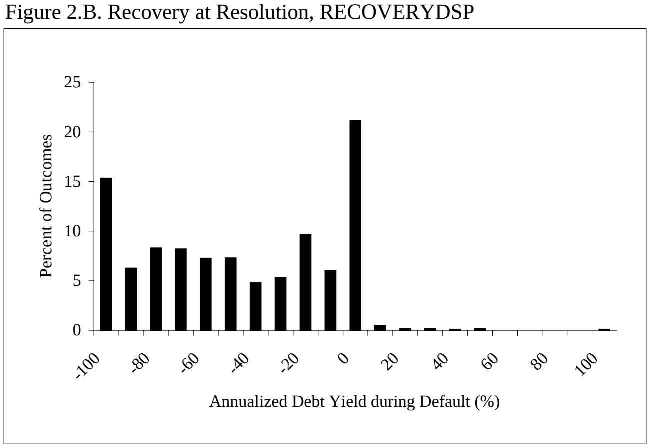

Figure 2.D. Recovery at Resolution, RECOVERYDST

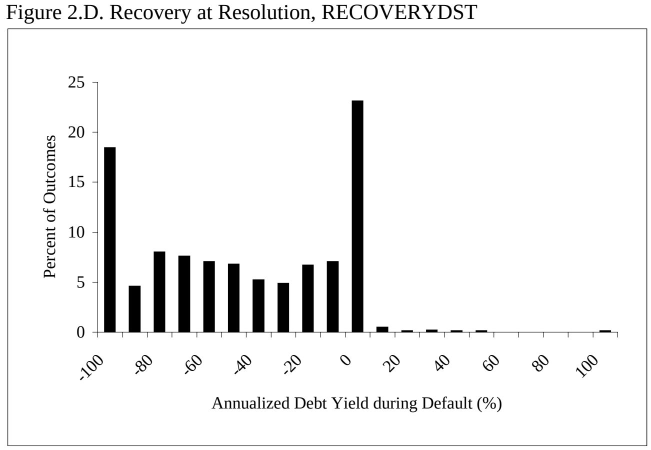

Figure 3. Distribution of Equity Yields during Default

Equity yields are from CRSP Daily Returns database. Yields are annualized for comparison across different default durations. FIRMEQUITYRETURNC2R is measured from the date of last cash payment to resolution of the default. FIRMEQUITYRETURNC2R is measured from the date of default to resolution.

Figure 3.A. Distribution of Equity Yields during Default: FIRMEQUITYRETURNC2R

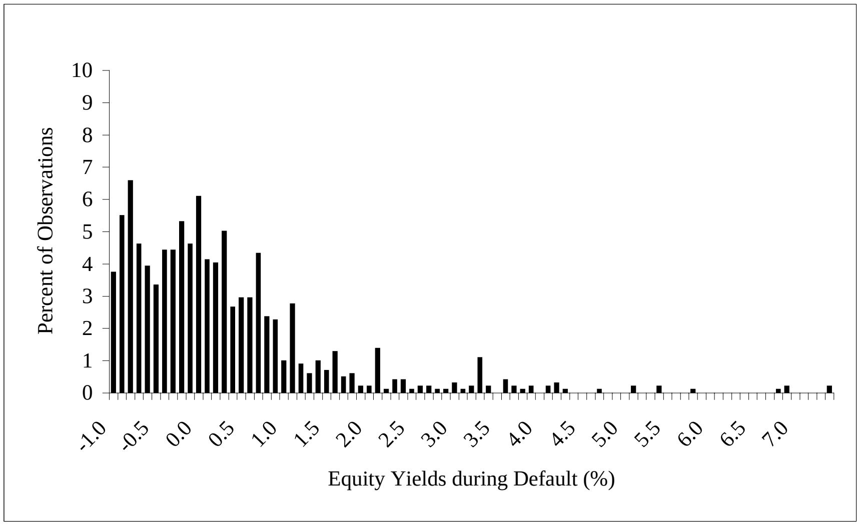

Figure 3.B. Distribution of Equity Yields during Default: FIRMEQUITYRETURND2R

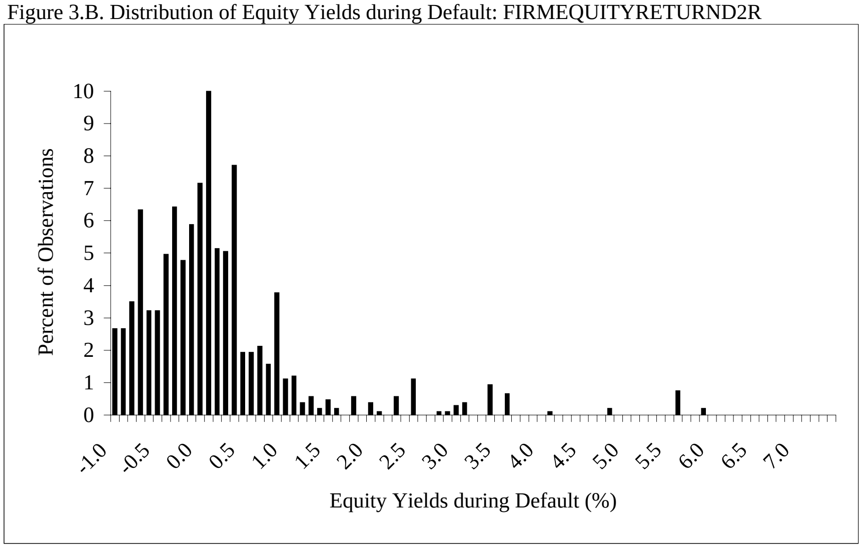

Figure 4. Propensity for Equity De-listing during Default

Equity de-listings are from CRSP Daily Returns database. Dates of default events are from Standard & Poor's LossStats™ Database. Dates of de-listing, re-listing, and default resolution are used to compute number of days listed during default.

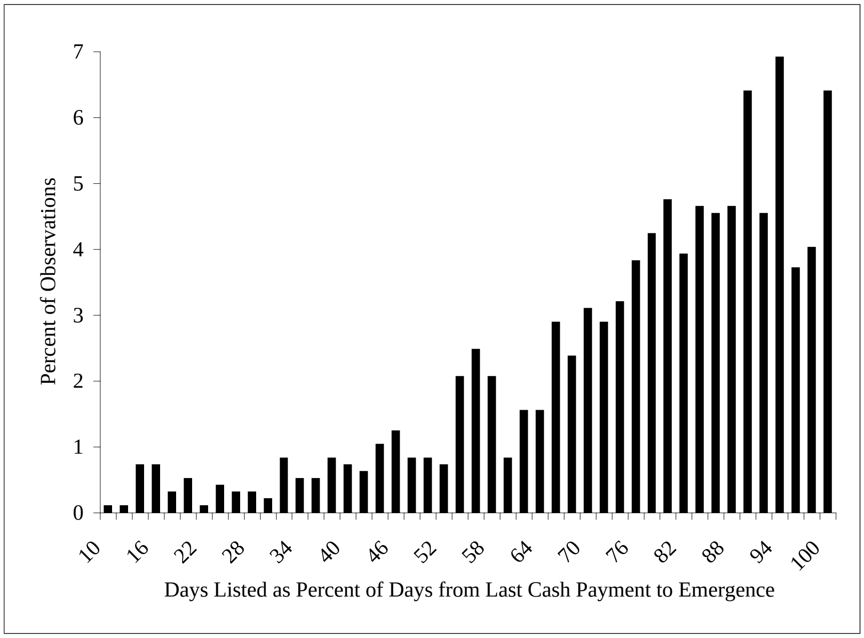

Table I

Panel A: Sample Summary Statistics and Variable Descriptions

CRSP refers to the CRSP Daily Returns database. LossStats is the Standard & Poor's LossStats[TM] Database. Compustat is the Standard & Poor's Compustat Annual data. Sample presented in table is the maximum number of observations for the least restrictive models, that is, after screening for δ, σ, LEVERAGE, ORIGINALMATURITY, SECURED, and remaining listed more than 50% of days in default.

Variable Name	N	MEAN	STD	MIN	MAX	MEDIAN	Definition	Source
Dependent Variables:								
DAYSC2R	663	572.706	331.594	3.000	2603.000	529.000	Days from Last Cash Payment to Resolution	LossStats
DAYSD2R	662	474.178	313.160	0.000	2472.000	405.000	Days from Default to Resolution	LossStats
DAYSB2R	247	304.559	258.850	0.000	1741.000	245.000	Days from Bankruptcy to Resolution	LossStats
RECOVERYNSP	663	-45.784	41.276	-100.000	77.465	-52.164	Nominal Debt Yield during Default, S&P Preferred Valuation Method	LossStats
RECOVERYDSP	663	-51.199	37.040	-100.000	42.886	-56.544	Discounted Debt Yield during Default, S&P Preferred Valuation Method	LossStats
RECOVERYNST	447	-45.095	42.022	-100.000	58.425	-48.824	Nominal Debt Yield during Default, Settlement Price Valuation Method	LossStats
RECOVERYDST	447	-49.881	38.787	-100.000	42.886	-55.809	Discounted Debt Yield during Default, Settlement Price Valuation Method	LossStats
FIRMEQUITYRETURNC2R	650	0.267	1.316	-0.999	7.430	-0.028	Company Equity Yield during Default, Last Cash Payment to Resolution	CRSP
FIRMEQUITYRETURND2R	649	0.227	1.169	-0.999	5.946	0.020	Company Equity Yield during Default, Default to Resolution	CRSP
Independent Variables:								
δ	663	0.869	0.495	-1.966	1.412	0.955	CRSP VW Index Return minus Firm Compounded Return for Year Prior to Event	CRSP
σ	663	0.296	0.183	0.036	1.488	0.248	Standard Deviation of Monthly Firm Returns for Year Prior to Event	CRSP
δ*RECESSION	663	0.198	0.381	-1.696	1.139	0.000	δ*Recession	CRSP and NBER
σ*RECESSION	663	0.094	0.208	0.000	1.488	0.000	σ*Recession	CRSP and NBER
PERATIO	444	-0.192	3.424	-18.182	25.125	-0.055	Company PE Ratio at Event	Compustat
LEVERAGE	663	0.225	0.304	0.000	0.996	0.000	Proportion of Debt Principal Senior to Defaulted Debt	LossStats
ORIGINALMATURITY	663	8.926	5.979	0.033	30.047	7.575	Original Maturity of Defaulted Debt	LossStats
SECURED	663	0.389	0.488	0.000	1.000	0.000	Dummy Variable Equals One if Defaulted Debt is Collateralized	LossStats
CRSPVWC2R	655	0.312	0.343	-0.454	1.990	0.285	CRSP Value-Weighted Index Return from Last Cash Payment to Resolution	CRSP
CRSPVWD2R	649	0.266	0.324	-0.454	1.814	0.255	CRSP Value-Weighted Index Return from Default to Resolution	CRSP
BONDRATING	663	29.054	3.614	9.000	30.000	30.000	Company Bond Rating at Event (Numerical)	LossStats
DIVADJRFR	663	4.800	2.192	-6.317	9.270	4.860	Current Dividend-adjusted Risk-free Rate using U.S. 5-yr T-Note YTM	Compustat
DIVYLD	663	0.668	3.533	0.000	50.000	0.000	Company Dividend Yield at Event	Compustat
5YRTREASYTM	663	5.913	1.549	2.900	9.510	5.700	U.S. 5-yr Treasury Note YTM	Compustat
INDBETA	203	1.475	3.278	-21.680	6.400	1.274	Industry Average Beta at Event	Compustat
FIRMBETA	300	1.179	1.312	-2.228	4.676	1.077	Company Beta at Event	Compustat
INDPERATIO	282	4.220	12.604	-41.187	40.974	3.400	Industry PE Ratio at Event	Compustat
DEBTPRIORITY	663	1.659	0.838	1.000	5.000	1.000	Priority Ranking of Debt Prior to Default based on Legal Contract	LossStats
$PRICE_{t+30}$	381	38.776	28.766	0.500	112.000	32.950	Price of Debt 30 Days after Default (Percent of Par)	LossStats
MATDEF	646	5.669	4.568	0.000	24.452	5.003	Remaining Maturity of Debt at Default	LossStats
PERCENTJUNIOR	663	0.218	0.287	0.000	0.995	0.000	Percent of Debt Principal Junior to Defaulted Debt	LossStats
RECESSION	663	0.258	0.438	0.000	1.000	0.000	NBER Recessions: Jul 1981-Nov 1982; Jul 190-Mar 1991; Mar 2001-Nov 2001	NBER

Table I

Panel B: Correlation Coefficients

The following table contains the Pearson product-moment correlations among the variables used in the various ecnometric specifications. Sample presented in table is the maximum number of observations for the least restrictive models, that is, after screening for δ, σ, LEVERAGE, ORIGINALMATURITY, SECURED, and remaining listed more than 50% of days in default. Variable definitions and other summary statitstics are given in Table I, Panel A.

Variable Name	DAYSC2R	DAYSD2R	DAYSB2R	RECOVERYNSP	RECOVERYDSP	RECOVERYNST	RECOVERYDST	FIRMEQUITYRETURNC2R	FIRMEQUITYRETURND2R	δ	σ	δ*RECESSION	σ*RECESSION	PERATIO	LEVERAGE	ORIGINALMATURITY	SECURED	CRSPVWC2R	CRSPVWD2R	BONDRATING	DIVADJRFR	DIVYLD	5YRTREASYTM	INDBETA	FIRMBETA	INDPERATIO	DEBTPRIORITY	$PRICE_{t+30}$	MATDEF	PERCENTJUNIOR	RECESSION
Dependent Variables:																															
DAYSC2R	1	0.979	0.698	0.153	0.157	0.137	0.130	-0.033	-0.010	0.111	-0.172	0.052	-0.039	0.039	0.186	0.189	-0.170	0.436	0.484	0.026	0.048	0.110	0.391	-0.020	-0.279	0.350	0.236	-0.219	0.161	-0.170	0.009
DAYSD2R	0.979	1	0.715	0.179	0.184	0.159	0.154	-0.029	0.002	0.133	-0.189	0.040	-0.058	0.026	0.150	0.141	-0.092	0.428	0.496	0.032	0.055	0.114	0.391	-0.033	-0.239	0.358	0.196	-0.182	0.129	-0.105	-0.003
DAYSB2R	0.698	0.715	1	-0.075	-0.055	-0.028	-0.023	-0.004	0.021	0.068	-0.125	0.003	-0.058	0.048	0.153	0.080	-0.050	0.254	0.225	-0.095	0.066	0.205	0.156	-0.010	-0.021	0.160	0.094	-0.158	0.087	-0.097	-0.005
RECOVERYNSP	0.153	0.179	-0.075	1	0.990	0.984	0.981	-0.169	-0.160	0.089	-0.154	-0.102	-0.101	-0.058	-0.383	-0.306	0.470	-0.063	0.003	-0.108	0.002	0.098	0.128	-0.158	0.030	0.024	-0.349	0.599	-0.277	0.441	-0.097
RECOVERYDSP	0.157	0.184	-0.055	0.990	1	0.983	0.983	-0.161	-0.155	0.091	-0.145	-0.099	-0.094	-0.058	-0.388	-0.302	0.473	-0.059	0.007	-0.113	-0.020	0.110	0.092	-0.151	0.034	0.046	-0.358	0.608	-0.277	0.444	-0.091
RECOVERYNST	0.137	0.159	-0.028	0.984	0.983	1	0.998	-0.145	-0.109	0.083	-0.105	-0.132	-0.075	0.027	-0.424	-0.301	0.531	-0.006	0.039	-0.101	0.029	0.029	0.051	-0.139	-0.184	0.009	-0.378	0.564	-0.239	0.508	-0.104
RECOVERYDST	0.130	0.154	-0.023	0.981	0.983	0.998	1	-0.140	-0.104	0.083	-0.098	-0.136	-0.074	0.028	-0.430	-0.304	0.543	0.001	0.048	-0.104	0.024	0.032	0.029	-0.131	-0.180	0.015	-0.386	0.573	-0.243	0.518	-0.107
FIRMEQUITYRETURNC2R	-0.033	-0.029	-0.004	-0.169	-0.161	-0.145	-0.140	1	0.857	-0.023	0.064	0.024	-0.041	0.078	0.018	0.150	-0.092	0.179	0.139	0.029	0.036	-0.070	-0.067	0.041	0.119	-0.127	0.004	-0.167	0.085	0.042	-0.021
FIRMEQUITYRETURND2R	-0.010	0.002	0.021	-0.160	-0.155	-0.109	-0.104	0.857	1	0.022	-0.011	0.060	-0.016	0.079	0.074	0.104	-0.080	0.109	0.123	0.026	0.018	-0.060	-0.080	0.020	-0.176	-0.064	0.020	-0.183	0.066	0.039	0.033
Independent Variables:																															
δ	0.111	0.133	0.068	0.089	0.091	0.083	0.083	-0.023	0.022	1	-0.215	0.064	-0.211	-0.035	0.173	-0.102	0.089	-0.076	-0.037	-0.058	0.151	-0.033	0.285	-0.036	0.094	-0.161	0.197	-0.054	0.015	0.117	-0.120
σ	-0.172	-0.189	-0.125	-0.154	-0.145	-0.105	-0.098	0.064	-0.011	-0.215	1	0.021	0.635	-0.058	-0.085	-0.108	0.053	0.026	-0.083	-0.024	-0.069	-0.087	-0.318	0.115	0.001	-0.100	-0.098	-0.036	-0.113	0.049	0.222
δ*RECESSION	0.052	0.040	0.003	-0.102	-0.099	-0.132	-0.136	0.024	0.060	0.064	0.021	1	0.526	-0.184	-0.029	-0.012	-0.145	-0.101	-0.106	0.098	-0.054	0.230	-0.049	0.082	-0.152	-0.063	-0.054	-0.185	0.013	-0.085	0.883
σ*RECESSION	-0.039	-0.058	-0.058	-0.101	-0.094	-0.075	-0.074	-0.041	-0.016	-0.211	0.635	0.526	1	-0.084	-0.068	-0.086	-0.043	-0.023	-0.092	0.104	-0.062	0.073	-0.184	0.173	-0.199	0.016	-0.108	-0.071	-0.067	-0.027	0.769
PERATIO	0.039	0.026	0.048	-0.058	-0.058	0.027	0.028	0.078	0.079	-0.035	-0.058	-0.184	-0.084	1	0.027	0.002	0.072	0.258	0.250	-0.015	-0.116	-0.099	0.064	-0.111	-0.103	0.146	0.111	-0.092	0.045	0.098	-0.175
LEVERAGE	0.186	0.150	0.153	-0.383	-0.388	-0.424	-0.430	0.018	0.074	0.173	-0.085	-0.029	-0.068	0.027	1	0.355	-0.547	0.024	0.014	-0.003	0.102	-0.062	0.263	-0.036	0.056	-0.064	0.814	-0.383	0.297	-0.452	-0.063
ORIGINALMATURITY	0.189	0.141	0.080	-0.306	-0.302	-0.301	-0.304	0.150	0.104	-0.102	-0.108	-0.012	-0.086	0.002	0.355	1	-0.483	0.100	0.121	-0.091	0.085	0.005	0.243	-0.085	-0.046	0.086	0.407	-0.286	0.837	-0.369	-0.051
SECURED	-0.170	-0.092	-0.050	0.470	0.473	0.531	0.543	-0.092	-0.080	0.089	0.053	-0.145	-0.043	0.072	-0.547	-0.483	1	-0.044	-0.008	-0.041	-0.047	-0.089	-0.119	-0.035	0.050	-0.054	-0.573	0.472	-0.382	0.620	-0.096
CRSPVWC2R	0.436	0.428	0.254	-0.063	-0.059	-0.006	0.001	0.179	0.109	-0.076	0.026	-0.101	-0.023	0.258	0.024	0.100	-0.044	1	0.923	-0.063	0.045	-0.059	0.093	-0.118	-0.153	0.117	0.025	-0.221	0.078	-0.057	-0.096
CRSPVWD2R	0.484	0.496	0.225	0.003	0.007	0.039	0.048	0.139	0.123	-0.037	-0.083	-0.106	-0.092	0.250	0.014	0.121	-0.008	0.923	1	-0.118	0.075	-0.031	0.148	-0.167	-0.082	0.214	0.028	-0.187	0.103	-0.033	-0.123
BONDRATING	0.026	0.032	-0.095	-0.108	-0.113	-0.101	-0.104	0.029	0.026	-0.058	-0.024	0.098	0.104	-0.015	-0.003	-0.091	-0.041	-0.063	-0.118	1	-0.035	0.013	-0.007	0.341	-0.204	0.215	-0.009	-0.181	-0.068	-0.034	0.119
DIVADJRFR	0.048	0.055	0.066	0.002	-0.020	0.029	0.024	0.036	0.018	0.151	-0.069	-0.054	-0.062	-0.116	0.102	0.085	-0.047	0.045	0.075	-0.035	1	-0.232	0.413	0.017	-0.094	-0.048	0.124	-0.072	0.109	-0.013	-0.071
DIVYLD	0.110	0.114	0.205	0.098	0.110	0.029	0.032	-0.070	-0.060	-0.033	-0.087	0.230	0.073	-0.099	-0.062	0.005	-0.089	-0.059	-0.031	0.013	-0.232	1	-0.008	0.120	0.077	-0.018	-0.012	0.099	0.025	-0.080	0.236
5YRTREASYTM	0.391	0.391	0.156	0.128	0.092	0.051	0.029	-0.067	-0.080	0.285	-0.318	-0.049	-0.184	0.064	0.263	0.243	-0.119	0.093	0.148	-0.007	0.413	-0.008	1	0.074	-0.155	0.071	0.378	-0.134	0.252	-0.003	-0.118
INDBETA	-0.020	-0.033	-0.010	-0.158	-0.151	-0.139	-0.131	0.041	0.020	-0.036	0.115	0.082	0.173	-0.111	-0.036	-0.085	-0.035	-0.118	-0.167	0.341	0.017	0.120	0.074	1	-0.247	0.046	-0.035	-0.070	-0.005	0.040	0.151
FIRMBETA	-0.279	-0.239	-0.021	0.030	0.034	-0.184	-0.180	0.119	-0.176	0.094	0.001	-0.152	-0.199	-0.103	0.056	-0.046	0.050	-0.153	-0.082	-0.204	-0.094	0.077	-0.155	-0.247	1	-0.290	0.078	0.194	-0.022	0.042	-0.201
INDPERATIO	0.350	0.358	0.160	0.024	0.046	0.009	0.015	-0.127	-0.064	-0.161	-0.100	-0.063	0.016	0.146	-0.064	0.086	-0.054	0.117	0.214	0.215	-0.048	-0.018	0.071	0.046	-0.290	1	-0.090	-0.128	0.063	-0.156	-0.038
DEBTPRIORITY	0.236	0.196	0.094	-0.349	-0.358	-0.378	-0.386	0.004	0.020	0.197	-0.098	-0.054	-0.108	0.111	0.814	0.407	-0.573	0.025	0.028	-0.009	0.124	-0.012	0.378	-0.035	0.078	-0.090	1	-0.335	0.383	-0.433	-0.098
$PRICE_{t+30}$	-0.219	-0.182	-0.158	0.599	0.608	0.564	0.573	-0.167	-0.183	-0.054	-0.036	-0.185	-0.071	-0.092	-0.383	-0.286	0.472	-0.221	-0.187	-0.181	-0.072	0.099	-0.134	-0.070	0.194	-0.128	-0.335	1	-0.337	0.347	-0.141
MATDEF	0.161	0.129	0.087	-0.277	-0.277	-0.239	-0.243	0.085	0.066	0.015	-0.113	0.013	-0.067	0.045	0.297	0.837	-0.382	0.078	0.103	-0.068	0.109	0.025	0.252	-0.005	-0.022	0.063	0.383	-0.337	1	-0.284	-0.024
PERCENTJUNIOR	-0.170	-0.105	-0.097	0.441	0.444	0.508	0.518	0.042	0.039	0.117	0.049	-0.085	-0.027	0.098	-0.452	-0.369	0.620	-0.057	-0.033	-0.034	-0.013	-0.080	-0.003	0.040	0.042	-0.156	-0.433	0.347	-0.284	1	-0.057
RECESSION	0.009	-0.003	-0.005	-0.097	-0.091	-0.104	-0.107	-0.021	0.033	-0.120	0.222	0.883	0.769	-0.175	-0.063	-0.051	-0.096	-0.096	-0.123	0.119	-0.071	0.236	-0.118	0.151	-0.201	-0.038	-0.098	-0.141	-0.024	-0.057	1

Table II

Individually-estimated Models of Debt Yields during Default

Below are individual estimates of the effects of discount rates (δ) and volatility (σ), their interactions with NBER recession periods, and LEVERAGE, ORIGINALMATURITY, and SECURED identification variables on various definitions of debt yields during default. Debt values are assumed to be par at default. Debt recoveries at resolution are from are from Standard & Poor's LossStats^{TM} Database. Yields are annualized for comparison across different default durations. RECOVERYNSP is the S&P Preferred recovery estimate (amount the Trading Price, Settlement Price, and Liquidity Event Price estimation methods) expressed in nominal terms. RECOVERYDSP is the S&P Preferred recovery estimate expressed in present-value discounted terms. RECOVERYNST is the S&P Settlement Price recovery estimate expressed in nominal terms. RECOVERYDST is the S&P Settlement Price recovery estimate expressed in present-value discounted terms. All present value discounts are computed using the original face coupon rate.

Results with *** are statistically significant at the 1% level, ** at the 5% level, and * at the 10% level.

	A			B			C			D		
Dependent Variable	RECOVERYNSP			RECOVERYDSP			RECOVERYNST			RECOVERYDST		
N	663			663			447			447		
R-Square	0.302			0.302			0.350			0.360		
Adj R-Sq	0.294			0.294			0.340			0.350		
	Coef	StdDev		Coef	StdDev		Coef	StdDev		Coef	StdDev	
Intercept	-29.312	5.518	***	-37.850	4.952	***	-40.028	6.527	***	-46.661	5.976	***
δ	6.074	3.011	**	5.940	2.702	**	8.190	3.371	***	7.501	3.087	***
σ	-64.343	10.901	***	-55.628	9.784	***	-56.161	12.070	***	-49.609	11.052	***
δ*RECESSION	-16.250	4.903	***	-14.420	4.401	***	-21.081	5.793	***	-19.500	5.304	***
σ*RECESSION	32.631	11.478	***	29.396	10.301	***	34.403	12.431	***	30.482	11.382	***
LEVERAGE	-29.051	5.623	***	-27.084	5.047	***	-30.777	6.789	***	-27.999	6.216	***
ORIGINALMATURITY	-0.660	0.263	***	-0.534	0.236	**	-0.004	0.316		0.029	0.289	
SECURED	25.455	3.700	***	22.959	3.321	***	33.076	4.749	***	31.845	4.348	***

Table III

Jointly-estimated Models of Equity and Debt Yields during Default

Below are estimates of the effects of discount rates (δ) and volatility (σ), their interactions with NBER recession periods, and LEVERAGE, ORIGINALMATURITY, and SECURED identifying variables on various definitions of debt returns during default jointly determined with the effects of the effects of discount rates (δ) and volatility (σ), their interactions with NBER recession periods, and the PERATIO identification variable on various definitions of equity returns during default. Debt values are assumed to be par at default. Debt recoveries at resolution are from are from Standard & Poor's LossStats^{TM} Database. Equity returns are from CRSP Daily Returns database. Yields during default are annualized for comparison across different default durations. RECOVERYNSP is the S&P Preferred recovery estimate (amount the Trading Price, Settlement Price, and Liquidity Event Price estimation methods) expressed in nominal terms. RECOVERYDSP is the S&P Preferred recovery estimate expressed in present-value discounted terms. RECOVERYNST is the S&P Settlement Price recovery estimate expressed in nominal terms. RECOVERYDST is the S&P Settlement Price recovery estimate expressed in present-value discounted terms. FIRMEQUITYRETURNC2R is measured from the date of last cash payment to resolution of the default. All present value discounts are computed using the original face coupon rate.

Results with *** are statistically significant at the 1% level, ** at the 5% level, and * at the 10% level.

	A			B			C			D		
Model Type	SHAREHOLDER			SHAREHOLDER			SHAREHOLDER			SHAREHOLDER		
Dependent Variable	FIRMEQUITYRETURNC2R			FIRMEQUITYRETURNC2R			FIRMEQUITYRETURNC2R			FIRMEQUITYRETURNC2R		
N	438			438			288			288		
R-Square	0.045			0.045			0.064			0.064		
Adj R-Sq	0.034			0.034			0.047			0.047		
	Coef	StdDev		Coef	StdDev		Coef	StdDev		Coef	StdDev	
INTERCEPT	0.034	0.189		0.034	0.189		0.087	0.213		0.087	0.213	
δ	-0.208	0.123	**	-0.208	0.123	**	-0.227	0.144	*	-0.227	0.144	*
σ	1.703	0.513	***	1.703	0.513	***	2.135	0.623	***	2.135	0.623	***
δ*RECESSION	0.529	0.227	**	0.529	0.227	**	0.679	0.286	***	0.679	0.286	***
σ*RECESSION	-2.028	0.497	***	-2.028	0.497	***	-2.563	0.584	***	-2.563	0.584	***
PERATIO	0.034	0.019	**	0.034	0.019	**	0.015	0.023		0.015	0.023	
Model Type	CREDITOR			CREDITOR			CREDITOR			CREDITOR		
Dependent Variable	RECOVERYNSP			RECOVERYDSP			RECOVERYNST			RECOVERYDST		
N	438			438			288			288		
R-Square	0.352			0.353			0.443			0.450		
Adj R-Sq	0.342			0.342			0.429			0.437		
	Coef	StdDev		Coef	StdDev		Coef	StdDev		Coef	StdDev	
INTERCEPT	-26.694	6.608	***	-33.100	6.094	***	-45.373	8.162	***	-50.334	7.526	***
δ	6.842	3.500	**	6.627	3.227	**	8.459	4.297	**	7.981	3.962	**
σ	-60.807	12.933	***	-55.130	11.927	***	-53.410	15.107	***	-49.097	13.930	***
δ*RECESSION	-23.324	5.830	***	-20.956	5.376	***	-29.696	7.132	***	-27.535	6.576	***
σ*RECESSION	37.893	12.617	***	34.638	11.635	***	35.694	14.623	***	32.699	13.484	***
LEVERAGE	-40.964	7.419	***	-38.272	6.842	***	-34.447	9.544	***	-31.807	8.801	***
ORIGINALMATURITY	-1.196	0.387	***	-1.097	0.357	***	-0.144	0.499		-0.110	0.460	
SECURED	21.715	4.325	***	19.961	3.988	***	38.908	5.809	***	36.849	5.357	***

Table IV

Jointly-estimated Models of Equity and Debt Yields during Default

Below are estimates of the effects of discount rates (δ) and volatility (σ), their interactions with NBER recession periods, the LEVERAGE, ORIGINALMATURITY, and SECURED identifying variables, and nine additional ad hoc control variables (Defined in Table I) on various definitions of debt returns during default jointly determined with the effects of the effects of discount rates (δ) and volatility (σ), their interactions with NBER recession periods, and the PERATIO identification variable on various definitions of equity returns during default. Debt values are assumed to be par at default. Debt recoveries at resolution are from are from Standard & Poor's LossStatsTM Database. Equity returns are from CRSP Daily Returns database. Yields during default are annualized for comparison across different default durations. RECOVERYNSP is the S&P Preferred recovery estimate (amount the Trading Price, Settlement Price, and Liquidity Event Price estimation methods) expressed in nominal terms. RECOVERYDSP is the S&P Preferred recovery estimate expressed in present-value discounted terms. RECOVERYNST is the S&P Settlement Price recovery estimate expressed in nominal terms. RECOVERYDST is the S&P Settlement Price recovery estimate expressed in present-value discounted terms. FIRMEQUITYRETURNC2R is measured from the date of last cash payment to resolution of the default. All present value discounts are computed using the original face coupon rate.

Results with *** are statistically significant at the 1% level, ** at the 5% level, and * at the 10% level.

	A			B			C			D		
Model Type	SHAREHOLDER			SHAREHOLDER			SHAREHOLDER			SHAREHOLDER		
Dependent Variable	FIRMEQUITYRETURNC2R			FIRMEQUITYRETURNC2R			FIRMEQUITYRETURNC2R			FIRMEQUITYRETURNC2R		
N	254			254			167			167		
R-Square	0.077			0.077			0.109			0.109		
Adj R-Sq	0.058			0.058			0.082			0.082		
	Coef	StdDev		Coef	StdDev		Coef	StdDev		Coef	StdDev	
INTERCEPT	0.149	0.229		0.149	0.229		0.200	0.227		0.200	0.227	
δ	-0.476	0.136	***	-0.476	0.136	***	-0.436	0.132	***	-0.436	0.132	***
σ	1.351	0.722	**	1.351	0.722	**	1.331	0.719	**	1.331	0.719	**
δ*RECESSION	0.798	0.288	***	0.798	0.288	***	1.072	0.322	***	1.072	0.322	***
σ*RECESSION	-1.914	0.637	***	-1.914	0.637	***	-1.775	0.623	***	-1.775	0.623	***
PERATIO	0.044	0.022	**	0.044	0.022	**	0.006	0.022		0.006	0.022	
Model Type	CREDITOR			CREDITOR			CREDITOR			CREDITOR		
Dependent Variable	RECOVERYNSP			RECOVERYDSP			RECOVERYNST			RECOVERYDST		
N	254			254			167			167		
R-Square	0.537			0.552			0.588			0.597		
Adj R-Sq	0.506			0.521			0.544			0.554		
	Coef	StdDev		Coef	StdDev		Coef	StdDev		Coef	StdDev	
INTERCEPT	43.743	20.998	**	34.756	18.991	**	32.014	24.502	*	23.085	22.470	
δ	10.276	4.004	***	9.975	3.622	***	11.819	4.749	***	10.950	4.355	***
σ	-82.995	17.161	***	-74.556	15.521	***	-98.044	18.720	***	-91.109	17.167	***
δ*RECESSION	-30.923	7.123	***	-27.477	6.443	***	-33.732	8.294	***	-31.343	7.606	***
σ*RECESSION	71.755	15.162	***	65.988	13.713	***	73.441	16.527	***	68.924	15.156	***
LEVERAGE	-5.773	13.855		-4.427	12.531		-10.682	19.467		-8.521	17.853	
ORIGINALMATURITY	1.195	0.738	*	1.145	0.668	**	1.898	0.996	**	1.759	0.913	**
SECURED	22.248	5.847	***	19.801	5.288	***	39.100	7.291	***	36.133	6.687	***
CRSPVWC2R	2.052	16.511		0.930	14.933		57.107	22.703	***	49.130	20.820	**
CRSPVWD2R	-7.998	18.463		-5.481	16.699		-75.439	25.183	***	-63.240	23.095	***
BONDRATING	-2.591	0.531	***	-2.496	0.480	***	-2.502	0.565	***	-2.380	0.518	***
DIVADJRFR	1.633	0.869	**	1.257	0.786	*	2.352	1.062	**	2.064	0.974	**
DIVYLD	4.710	0.924	***	4.520	0.835	***	1.906	1.809		1.524	1.659	
INDPERATIO	0.177	0.162		0.233	0.147	*	0.352	0.229	*	0.339	0.210	*
DEBTPRIORITY	-12.958	6.179	**	-12.416	5.589	**	-11.860	9.052	*	-11.122	8.302	*
MATDEF	-2.116	0.983	**	-1.963	0.889	**	-2.297	1.234	**	-2.118	1.131	**
PERCENTJUNIOR	21.257	8.154	***	21.190	7.375	***	9.956	10.589		11.721	9.710	

Table V

Jointly-estimated Models of Equity and Debt Yields during Default

Below are estimates of the effects of discount rates (δ) and volatility (σ), their interactions with NBER recession periods, the LEVERAGE, ORIGINALMATURITY, and SECURED identifying variables, and thirteen additional ad hoc control variables (Defined in Table I) on various definitions of debt returns during default jointly determined with the effects of the effects of discount rates (δ) and volatility (σ), their interactions with NBER recession periods, and the PERATIO identification variable on various definitions of equity returns during default. Debt values are assumed to be par at default. Debt recoveries at resolution are from are from Standard & Poor's LossStatsTM Database. Equity returns are from CRSP Daily Returns database. Yields during default are annualized for comparison across different default durations. RECOVERYNSP is the S&P Preferred recovery estimate (amount the Trading Price, Settlement Price, and Liquidity Event Price estimation methods) expressed in nominal terms. RECOVERYDSP is the S&P Preferred recovery estimate expressed in present-value discounted terms. RECOVERYNST is the S&P Settlement Price recovery estimate expressed in nominal terms. RECOVERYDST is the S&P Settlement Price recovery estimate expressed in present-value discounted terms. FIRMEQUITYRETURNC2R is measured from the date of last cash payment to resolution of the default. All present value discounts are computed using the original face coupon rate.

Results with *** are statistically significant at the 1% level, ** at the 5% level, and * at the 10% level.

	A			B			C			D		
Model Type	SHAREHOLDER			SHAREHOLDER			SHAREHOLDER			SHAREHOLDER		
Dependent Variable	FIRMEQUITYRETURNC2R			FIRMEQUITYRETURNC2R			FIRMEQUITYRETURNC2R			FIRMEQUITYRETURNC2R		
N	113			113			74			74		
R-Square	0.269			0.269			0.251			0.251		
Adj R-Sq	0.235			0.235			0.196			0.196		
	Coef	StdDev		Coef	StdDev		Coef	StdDev		Coef	StdDev	
INTERCEPT	0.809	0.379	**	0.809	0.379	**	1.569	0.580	***	1.569	0.580	***
δ	-0.704	0.207	***	-0.704	0.207	***	-1.134	0.306	***	-1.134	0.306	***
σ	0.445	1.088		0.445	1.088		0.603	1.101		0.603	1.101	
δ*RECESSION	2.014	0.473	***	2.014	0.473	***	1.440	0.490	***	1.440	0.490	***
σ*RECESSION	-1.144	0.877	*	-1.144	0.877	*	-2.050	0.857	***	-2.050	0.857	***
PERATIO	0.476	0.088	***	0.476	0.088	***	1.469	0.480	***	1.469	0.480	***
Model Type	CREDITOR			CREDITOR			CREDITOR			CREDITOR		
Dependent Variable	RECOVERYNSP			RECOVERYDSP			RECOVERYNST			RECOVERYDST		
N	113			113			74			74		
R-Square	0.801			0.806			0.857			0.857		
Adj R-Sq	0.758			0.763			0.803			0.803		
	Coef	StdDev		Coef	StdDev		Coef	StdDev		Coef	StdDev	
INTERCEPT	-67.110	31.129	**	-65.830	28.591	**	-17.343	43.608		-18.946	40.410	
δ	26.604	5.995	***	24.283	5.506	***	32.391	11.302	***	29.123	10.473	***
σ	-57.228	17.284	***	-53.314	15.875	***	-73.490	27.528	***	-68.576	25.509	***
δ*RECESSION	-41.481	9.412	***	-38.597	8.644	***	-46.737	20.547	**	-42.635	19.040	**
σ*RECESSION	61.029	14.837	***	56.649	13.627	***	47.118	24.320	**	43.363	22.537	**
LEVERAGE	9.488	17.626		7.530	16.189		45.672	23.508	**	42.046	21.784	**
ORIGINALMATURITY	-0.606	0.719		-0.569	0.660		-0.279	1.062		-0.321	0.984	
SECURED	-13.768	8.419	*	-11.881	7.732	*	-7.350	13.038		-7.258	12.082	
CRSPVWC2R	13.018	18.765		8.539	17.234		22.482	27.988		17.598	25.936	
CRSPVWD2R	23.210	24.221		24.730	22.246		-69.491	40.331	**	-62.477	37.373	**
BONDRATING	-1.454	0.613	***	-1.407	0.563	***	0.419	0.775		0.373	0.718	
DIVADJRFR	2.080	1.105	**	1.711	1.015	**	5.637	1.401	***	4.933	1.299	***
DIVYLD	1.950	1.677		1.946	1.540		7.643	5.307	*	7.124	4.918	*
5YRTREASYTM	4.199	4.102		3.658	3.768		-6.162	6.464		-6.272	5.990	
INDBETA	-0.432	0.652		-0.284	0.599		-0.547	0.901		-0.390	0.834	
FIRMBETA	2.626	2.219		2.498	2.038		-4.952	4.906		-4.870	4.546	
INDPERATIO	0.462	0.229	**	0.410	0.210	**	0.626	0.308	**	0.537	0.286	**
DEBTPRIORITY	-15.697	8.363	**	-13.969	7.681	**	-35.176	12.241	***	-31.714	11.343	***
$PRICE_{t+30}$	1.111	0.176	***	1.019	0.162	***	1.008	0.240	***	0.937	0.222	***
MATDEF	0.813	0.914		0.760	0.839		0.732	1.217		0.722	1.128	
PERCENTJUNIOR	-15.152	16.979		-14.940	15.595		-4.784	30.138		-2.235	27.928	

Figure B1. Duration of Default Events

Default events are from Standard & Poor's LossStats™ Database, which contains a total of 1,706 missed cash payments; 1,808 defaults (more than one instrument of a firm can default from a missed cash payment); and 602 bankruptcies between 1987 and 2003. DAYSC2R are days from last cash payment to resolution; DAYSD2R are days from default to resolution; and DAYSB2R are days from bankruptcy to resolution.

Figure B1.A: Days from Last Cash Payment to Resolution, DAYSC2R

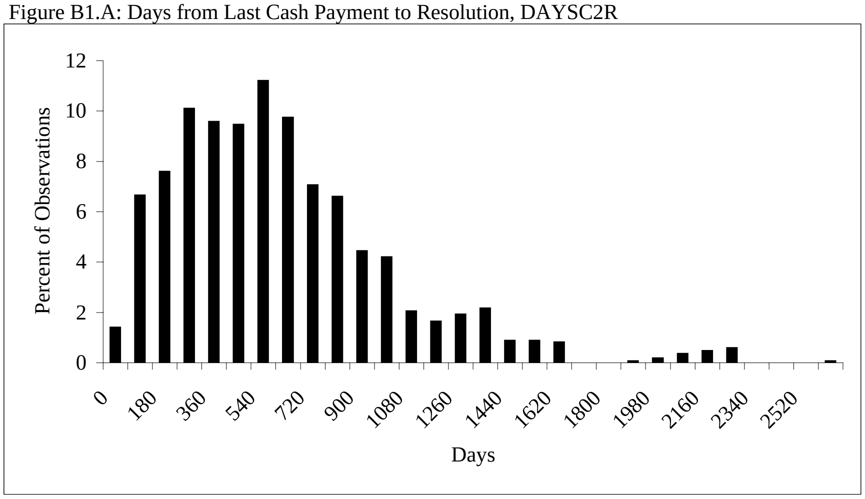

Figure B1.B: Days from Default to Resolution, DAYSD2R

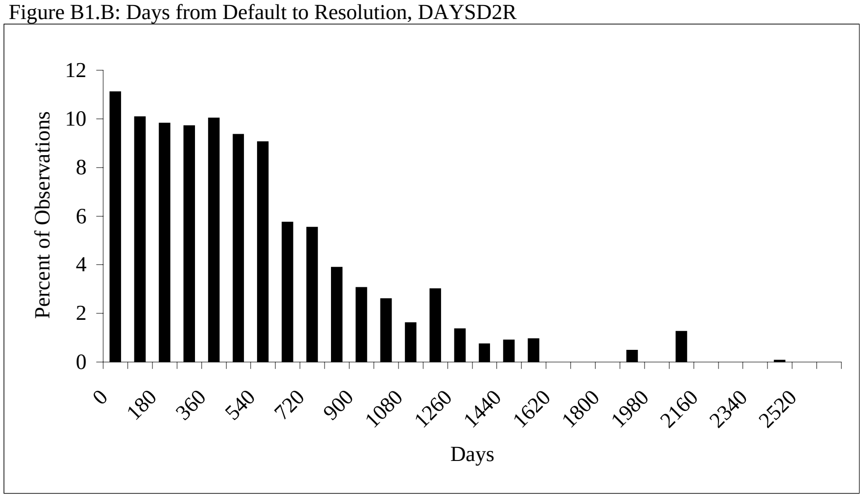

Figure B1.C: Days from Bankruptcy to Resolution, DAYSB2R

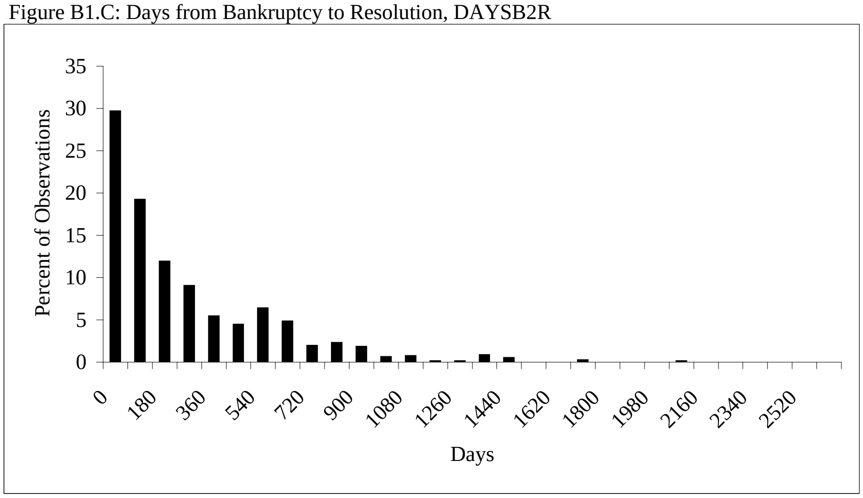

Table BI

Individually-estimated Models of Days in Default

Below are individual estimates of the effects of discount rates (δ) and volatility (σ), their interactions with NBER recession periods, and LEVERAGE, ORIGINALMATURITY, and SECURED identifying variables on days in various definitions of default. Dates of the different default events and resolution are from Standard & Poor's LossStats™ Database. DAYSC2R are days from last cash payment to resolution; DAYSD2R are days from default to resolution; and DAYSB2R are days from bankruptcy to resolution.

Results with *** are statistically significant at the 1% level, ** at the 5% level, and * at the 10% level.

	A			B			C		
Dependent Variable	DAYSC2R			DAYSD2R			DAYSB2R		
N	663			662			247		
R-Square	0.092			0.078			0.042		
Adj R-Sq	0.082			0.069			0.014		
	Coef	StdDev		Coef	StdDev		Coef	StdDev	
Intercept	548.761	50.554	***	441.398	48.419	***	256.465	74.105	***
δ	69.461	27.588	***	71.125	26.453	***	40.995	48.584	
σ	-414.195	99.875	***	-421.432	95.023	***	-143.765	121.528	
δ*RECESSION	-34.647	44.926		-32.347	42.739		40.723	82.758	
σ*RECESSION	259.632	105.159	***	241.020	100.069	***	-59.114	180.363	
LEVERAGE	86.540	51.516	**	90.133	49.015	**	126.468	66.973	**
ORIGINALMATURITY	7.273	2.414	***	6.132	2.299	***	1.350	3.885	
SECURED	-40.284	33.897		11.089	32.276		17.198	47.704	

Table BII

Jointly-estimated Models of Equity Yields and Days in Default

Below are estimates of the effects of discount rates (δ) and volatility (σ), their interactions with NBER recession periods, and LEVERAGE, ORIGINALMATURITY, and SECURED identifying variables on days in various definitions of default jointly determined with the effects of the effects of discount rates (δ) and volatility (σ), their interactions with NBER recession periods, and the PERATIO identification variable on various definitions of equity returns during default. Dates of the different default events and resolution are from Standard & Poor's LossStats™ Database. DAYSC2R are days from last cash payment to resolution; DAYSD2R are days from default to resolution; and DAYSB2R are days from bankruptcy to resolution. Equity returns are from CRSP Daily Returns database. Returns are annualized for comparison across different default durations. FIRMEQUITYRETURNC2R is measured from the date of last cash payment to resolution of the default. FIRMEQUITYRETURND2R is measured from the date of default to resolution.

Results with *** are statistically significant at the 1% level, ** at the 5% level, and * at the 10% level.

	A			B			C		
Model Type	SHAREHOLDER			SHAREHOLDER			SHAREHOLDER		
Dependent Variable	FIRMEQUITYRETURNC2R			FIRMEQUITYRETURNC2R			FIRMEQUITYRETURND2R		
N	438			438			440		
R-Square	0.045			0.045			0.020		
Adj R-Sq	0.034			0.034			0.009		
	Coef	StdDev		Coef	StdDev		Coef	StdDev	
INTERCEPT	0.034	0.189		0.034	0.189		0.143	0.182	
δ	-0.208	0.123	**	-0.208	0.123	**	0.000	0.119	
σ	1.703	0.513	***	1.703	0.513	***	0.459	0.497	
δ*RECESSION	0.529	0.227	**	0.529	0.227	**	0.509	0.217	**
σ*RECESSION	-2.028	0.497	***	-2.028	0.497	***	-0.783	0.480	*
PERATIO	0.034	0.019	**	0.034	0.019	**	0.037	0.018	**
Model Type	CREDITOR			CREDITOR			CREDITOR		
Dependent Variable	DAYSC2R			DAYSD2R			DAYSD2R		
N	438			438			440		
R-Square	0.098			0.086			0.086		
Adj R-Sq	0.084			0.071			0.072		
	Coef	StdDev		Coef	StdDev		Coef	StdDev	
INTERCEPT	571.170	54.355	***	462.984	51.613	***	454.018	51.414	***
δ	99.999	28.785	***	99.697	27.333	***	99.617	27.135	***
σ	-498.598	106.376	***	-485.561	101.010	***	-463.737	100.609	***
δ*RECESSION	-155.424	47.951	***	-145.041	45.532	***	-151.488	44.807	***
σ*RECESSION	385.108	103.776	***	358.814	98.541	***	340.687	97.809	***
LEVERAGE	-50.176	61.020		-33.794	57.942		-29.510	57.522	
ORIGINALMATURITY	7.628	3.184	***	5.751	3.023	**	6.359	3.022	**
SECURED	-80.929	35.572	**	-27.768	33.777		-25.165	33.414	